Recall the poll of 1207 people of whom 53% thought Obama won the third debate. Is it reasonable that this is random variation from an even split?

1) \( H_0: \mu = 0.5 \) where \( \mu \) is the population proportion that think Obama won the third debate

2) \( \alpha = 0.05 \)

3) Recall that if \( X \sim \text{Binomial}(n, \mu) \) then by CLT \( \hat{\mu} \) is approximately normally distributed with \( \mu \) and std dev \( \sqrt{\frac{\mu(1-\mu)}{n}} \)

So the test statistic is \( z = \frac{\hat{\mu} - \mu}{\sqrt{\frac{\mu(1-\mu)}{n}}} \)

Sampling distribution is the standard normal

Note: for a hypothesis test, use the hypothesized value of \( \mu \) in the std dev, not \( \hat{\mu} \) (like in a CI)

4) \( z = \frac{0.53 - 0.5}{\sqrt{(0.5)(0.5)/1207}} = 2.08 \)

a) Critical region: reject if \( z < -1.96 \) or \( z > 1.96 \)

b) \( \text{p-value} = P(|Z| > 2.08) = P(Z < -2.08) + P(Z > 2.08) = 2P(Z < -2.08) = 2(0.0188) = 0.0376 \)

5) Reject \( H_0 \)

6) Conclude it is not reasonable that 50% of the people think Obama won the third debate — not an even split.

Note: 3 equivalent ways to reach the same conclusion:

1) Is \( \mu \) outside the confidence interval?
2) Is the test statistic in the critical region?
3) Is the \( \text{p-value} \) less than the significance level (.05)?
Two-Sample Tests

Have the average number of chips per cookie changed over time?

We need to take into account that there is a lot of variability between cookies in each sample.

Let \( M_1 \) = population mean # chips per cookie in 2012
\( M_2 \) = popn mean # chips per cookie in 2008

\( \bar{X}_1 \) = sample mean in 2012
\( \bar{X}_2 \) = sample mean in 2008

\( \sigma_1 \) and \( \sigma_2 \) are popn standard deviation in 2012 and 2008
\( s_1 \) and \( s_2 \) are sample std. dev.'s

\( n_1 \) and \( n_2 \) are sample sizes

By CLT \( \bar{X}_1 \) and \( \bar{X}_2 \) are approximately normally distributed
with means \( M_1 \) and \( M_2 \) and std dev's \( \frac{\sigma_1}{\sqrt{n_1}} \) and \( \frac{\sigma_2}{\sqrt{n_2}} \)

We want to test if \( M_1 = M_2 \). The sum (or difference) of two normals is also normal. So \( \bar{X}_1 - \bar{X}_2 \) is approx. normal

with mean \( M_1 - M_2 \) and std dev \( \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \)

In practice, \( \sigma_1 \) and \( \sigma_2 \) are unknown so estimate them with \( s_1 \) and \( s_2 \)

Test statistic is

\[
T = \frac{(\bar{X}_1 - \bar{X}_2) - (M_1 - M_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}
\]

How many df? If we assume \( \sigma_1 = \sigma_2 \), then df = \( n_1 + n_2 - 2 \)
Otherwise, use the smaller of \( (n_1 - 1) \) and \( (n_2 - 1) \)

JMP will estimate with a more complicated formula.

Key idea: we are comparing two populations to each other, without testing a particular value.

Testing \( M_1 = M_2 \) not \( M_1 = M_2 = 20 \).
$\overline{x}_1 = 20.39 \quad \overline{x}_2 = 19.89$

$s_1 = 4.96 \quad s_2 = 4.29$

$n_1 = 312 \quad n_2 = 183$

6 steps

1) $H_0: \mu_1 = \mu_2$  \quad where $\mu_1$ is pop'n mean #chips in 2012

$H_1: \mu_1 \neq \mu_2$  \quad $\mu_2$ is pop'n mean #chips in 2008

2) $\alpha = 0.05$

3) $t = \frac{\overline{x}_1 - \overline{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$  \quad $\mu_1 - \mu_2 = 0$

   Sampling dist'n is $t$ with $df = \min(n_1-1, n_2-1)$

4) Critical region

   $df = \min (312-1, 183-1) = \min (311, 182) = 182$

   reject if $t < -1.972$ or $t > 1.972$

   $t = \frac{20.39 - 19.89}{\sqrt{\frac{4.96^2}{312} + \frac{4.29^2}{183}}} = 1.18$

5) Fail to reject $H_0$

6) Conclude there is not enough evidence to show that the average number of chips per cookie has changed over time.

In 2007:  $\overline{x}_3 = 27.09 \quad s_3 = 4.44 \quad n_3 = 115$

   $df = 114$, reject if $t < -1.984$ or $t > 1.984$

   $t = \frac{20.39 - 27.09}{\sqrt{\frac{4.96^2}{312} + \frac{4.44^2}{115}}} = -13.39$

   Here we can conclude that the population mean #chips per cookie has changed.

95% CI for a difference in population means

$\overline{x}_1 - \overline{x}_2 \pm E \quad E = t_{\alpha/2} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \quad df = \min(n_1-1, n_2-1)$
Suppose a Fox poll after the third debate of 1002 people found that 48% thought Obama won.
Do we think these two polls are polling the same population?

6 steps

1) Let $p_1 =$ population proportion in the first poll who thought Obama won
   $p_2 =$ pop'n proportion in the second poll

   $H_0: p_1 = p_2$
   $H_1: p_1 \neq p_2$

2) $\alpha = .05$

3) Test statistic $Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{\hat{P}(1-\hat{P})}{n_1} + \frac{\hat{P}(1-\hat{P})}{n_2}}}$
   where $\hat{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{n_1 \hat{p}_1 + n_2 \hat{p}_2}{n_1 + n_2}$
   Combined sample proportion

4) $\hat{p} = \frac{1207(.53) + 1002(.48)}{1207 + 1002} = .507$

   $Z = \frac{.53 - .48}{\sqrt{.507(1-.507)(\frac{1}{1207} + \frac{1}{1002})}} = 2.34$

   a) Critical region is $Z < -1.96$ or $Z > 1.96$
   b) $p$-value = $P(Z < -2.34) + P(Z > 2.34) = 2P(Z < -2.34) = 2(.0096) = .0192$

5) Reject $H_0$ because $2.34 > 1.96$ or because $.0192 < .05$

6) Conclude not polling the same population.

A 95% CI for the difference in population proportions is

$\hat{p}_1 - \hat{p}_2 \pm z_{.025} \sqrt{\frac{\hat{P}(1-\hat{P})}{n_1} + \frac{\hat{P}(1-\hat{P})}{n_2}}$

$.53 - .48 \pm 1.96 \sqrt{.507(1-.507)(\frac{1}{1207} + \frac{1}{1002})} = .05 \pm .042$

$=.008, .092$

We are 95% confident that the true difference in population proportions is between .008 and .092.