Two sample t-tests

Assume working at a fast-food restaurant claims that drive-through customers spend more than walk-ups.

A random sample of 38 customers who come to the counter spent a mean of $5.19 with a std. dev. of $3.06. A random sample of 17 drive-through customers spent a mean of $5.94 with a std. dev. of $3.25. Test this claim.

\[
\bar{x}_1 = 5.19 \quad s_1 = 3.06 \quad n_1 = 38
\]
\[
\bar{x}_2 = 5.94 \quad s_2 = 3.25 \quad n_2 = 17
\]

1) Let \( \mu_1 \) = population mean expenditure of counter customers
\( \mu_2 \) = pop'n mean expenditure of drive-through customers

\( H_0: \mu_1 = \mu_2 \quad H_1: \mu_1 < \mu_2 \)

2) \( \alpha = 0.05 \)

3) \( t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s_1^2/n_1 + s_2^2/n_2}} \)

Sampling dist'n is \( t \) with \( df = \min(38-1, 17-1) = 16 \)

4) From table, one-tail 16 df .05, reject if \( t < -1.746 \)

\[
t = \frac{5.19 - 5.94}{\sqrt{\frac{3.06^2}{38} + \frac{3.25^2}{17}}} = -0.805
\]

5) Fail to reject \( H_0 \) \( -0.805 \not< -1.746 \)

6) There is not enough evidence to conclude that drive-through customers spend more than walk-ups.
Did Quiz 3 and Quiz 4 differ in difficulty?
In a class 155 people took both quizzes

<table>
<thead>
<tr>
<th>Quiz</th>
<th>mean</th>
<th>sd</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quiz 3</td>
<td>7.2</td>
<td>2.3</td>
</tr>
<tr>
<td>Quiz 4</td>
<td>7.7</td>
<td>2.4</td>
</tr>
</tbody>
</table>

1) Let $\mu_3 =$ pop'n mean of Quiz 3  
   $\mu_4 =$ pop'n mean of Quiz 4
   $H_0: \mu_3 = \mu_4$
   $H_1: \mu_3 \neq \mu_4$

2) $\alpha = 0.05$

3) $t = \frac{\bar{x}_3 - \bar{x}_4}{\sqrt{\frac{s_3^2}{n_3} + \frac{s_4^2}{n_4}}}$
   * with a 154 df

4) reject if $|t| > 1.97$
   From JMP, p-value is 0.063
   
   $t = \frac{\bar{x}_3 - \bar{x}_4}{\sqrt{\frac{2.3^2}{155} + \frac{2.4^2}{155}}} = -1.87$

5) fail to reject $H_0$

6) Conclude there is not enough evidence to show any difference

Is this the best we can do? Is there any additional information we should take into account? These are the same people taking both quizzes, need to control for student aptitude.

**Paired t-test**

When we want to compare two samples that are dependent, we should do a test for matched pairs.

We compute the differences $d_i = X_i - Y_i$. Then do a one-sample $t$ test to see if the population mean difference is 0.
Ex: We really have one sample of people who took 2 quizzes. For each person, compute \( d_i = \text{Quiz 3} - \text{Quiz 4} \)

We get \( \bar{d} = .56 \), \( s_d = 2.64 \)

1) \( \mu_d = \text{population mean difference in quiz scores} \)
   \[ H_0: \mu_d = 0 \]
   \[ H_1: \mu_d \neq 0 \]

2) \( \alpha = .05 \)

3) \[ t = \frac{\bar{d}}{s_d / \sqrt{n}} \]

4) Same critical region as before, reject \( H_0 \) if \( |t| > 1.97 \)

   \[ t = \frac{-0.56}{2.64 / \sqrt{155}} = -2.64 \]

   p-value = .009

5) reject \( H_0 \), \( -2.64 < -1.97 \)

6) Conclude the two quizzes are not equal in difficulty

p-value decreases and the conclusion changes because we used more information

We should do a paired test if and only if there is a reason to match the observations.

If the samples are dependent, \( s_d \) is typically smaller than the pooling of \( s_1 \) and \( s_2 \).
Comparing Population Variances  (Sec. 8.6)

Concept is important for ANOVA and regression

A survey of 65 beer drinkers ages 21-29 drank an average of 22.7 beers over the last month with a s.d. of 8.49 beers.
A survey of 107 beer drinkers ages 30-39 drank an average of 19.8 beers with a s.d. of 6.21 beers. Does the variability in beer consumption change with age?

1) \( \sigma^2_1 = \text{population variance of beer consumption by 21-29 year-olds} \)
   \( \sigma^2_2 = \text{pop'n variance of 30-39 year-olds} \)
   \[ H_0: \sigma^2_1 = \sigma^2_2 \quad H_1: \sigma^2_1 \neq \sigma^2_2 \]

2) \( \alpha = 0.05 \)

3) \( F = \frac{\hat{\sigma}^2_1}{\hat{\sigma}^2_2} \)  
   Sampling distribution is the F distribution with \( n_1-1 \) and \( n_2-1 \) df  
   Here \( 65-1 = 64 \) and \( 107-1 = 106 \) df

4) From table A-5, reject if \( F > 1.53 \) (use 60,120 df)

   \[ F = \frac{8.49^2}{6.20^2} = 1.87 \]

5) reject \( H_0 \) because \( 1.87 > 1.53 \)

6) Conclude that beer consumption variability does change with age.

This is the basis for ANOVA (Analysis of Variance)
Also used in multiple regression.