1. Complex numbers (10 points total)

(a) (3 points) Simplify $z = \frac{2}{1+i}$ (i.e. rearrange $z$ into standard form, like $z = a + ib$)

ANSWER:

(b) (3 points) Draw a plot of $z$ in an Argand diagram. Indicate on your Argand diagram above, what the modulus $r$ and the argument $\theta$ of $z$ represent.

ANSWER:

(c) (2 points) Calculate (or simply write down) the polar ($re^{i\theta}$) form of $z$, assuming that the principle argument of $\theta$ lies in the range $[-\pi, \pi]$.

ANSWER:

(d) (2 points) What is arg($z$) if the principle argument lies in the range $[0, 2\pi]$?

ANSWER:
2. **Linear systems and Applications** (20 points total)

This question concerns the traffic flow in a quaint English village as shown in the diagram above. The numbers of cars flowing into and out of the village (per hour) are known, but the flow between the three intersections is not known. There is no parking in the village.

(a) (2 points) Write the traffic flow as a linear system in the three unknowns \(x_1, x_2, x_3\), the flow rates between the junctions).

**ANSWER:**

(b) (2 points) Considering the system as a matrix equation \(Ax = b\), write down the augmented matrix \(M = [A|b]\) for this system. *(Hint: Be careful with the signs of the entries!)*

**ANSWER:**

(c) (4 points) Reduce the augmented matrix to *echelon form*. Circle the *pivots* in the result.

**ANSWER:**

(questio continued on next page)
(d) (2 points) What is the rank of $A$ and the rank of $M$?
   ANSWER:

(e) (1 point) Are the equations consistent? Explain your answer very briefly in terms of rank.
   ANSWER:

(f) (1 point) How many solutions are there to this system (0, 1, or $\infty$)? Explain your answer very briefly in terms of rank.
   ANSWER:

(g) (4 points) Solve the system for $(x_1, x_2, x_3)$ (setting any free variables to a parameter, e.g. $t$, if necessary). (Hint: You may continue to a full row reduced echelon form or back substitute, as you please.)
   ANSWER:

(h) (1 point) Are there any “real world” constraints on the free parameter $t$?
   ANSWER:

(i) (1 point) If I lived on the street going between B and C, and I like to play soccer in the street with my mates, what might be my preferred solution of this problem?
   ANSWER:

(j) (2 points) Considering the traffic flow diagram within the village as a directed graph (ignoring the unconnected roads in and out of the village), write down the adjacency matrix for the nodes A, B and C.
   ANSWER:
3. **Inverses of matrices** (14 points total)

   (a) (2 points) Is the coefficient matrix \( A \) in the traffic flow problem in Question 2 invertible? Justify your answer.

   ANSWER:

   (b) (8 points) Calculate the inverse of the following matrix

   \[
   A = \begin{pmatrix}
   -1 & 0 & 1 \\
   3 & -1 & -1 \\
   -1 & 1 & 0
   \end{pmatrix}
   \]

   ANSWER:
(c) (4 points) Using the inverse from part (b) above, solve the following system:

\[-x_1 + x_3 = 1\]
\[3x_1 - x_2 - x_3 = 1\]
\[-x_1 + x_2 = 1\]

ANSWER:
4. **MATLAB** (6 points total)

Write down the essential Matlab commands you would need to check your answers to Question 2 part (c) and Question 3 parts (b) and (c). Don’t forget to include the commands you need to input the matrices, etc.

ANSWER for Question 2(c) (2 points):

ANSWER for Question 3(b) (2 points):

ANSWER for Question 3(c) (2 points):