1 Multiple choice

1. How many solutions can a system of 3 equations in 5 unknowns have? Choose all that apply.
   A) Zero
   B) Exactly one
   C) Exactly two
   D) Infinitely many

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3. Consider an \( m \times n \) matrix, A. One of these statements is not like the others, one of them doesn’t belong! (which one?)
   A) \( \text{rref}(A) \) has a pivot position in every row.
   B) Each vector in \( \mathbb{R}^m \) can be written as a linear combination of the columns of \( A \).
   C) The rows of \( A \) span \( \mathbb{R}^m \).
   D) For each vector \( b \) in \( \mathbb{R}^m \), the equation \( Ax = b \) has a solution.

2 Written problems

1. Give an example of an equation that is not linear in the variables \( x_1, x_2, \) and \( x_3 \).

2. Compute the product \( Ax \), for the matrix and vector given below:

   \[
   A = \begin{bmatrix}
   1 & 1 & -1 \\
   -5 & 1 & 2 \\
   1 & -5 & 2
   \end{bmatrix}
   \quad x = \begin{bmatrix}
   1 \\
   1 \\
   2
   \end{bmatrix}
   \]

   What can you say about the columns of \( A \) from the product you just computed?
3. Suppose you arrive at the following reduced row-echelon augmented matrix while looking for the solution to $Ax = b$.

\[
\begin{bmatrix}
1 & 2 & 0 & 3 & 0 & \vdots & 2 \\
0 & 0 & 1 & -1 & 0 & \vdots & 2 \\
0 & 0 & 0 & 0 & 1 & \vdots & -2 \\
0 & 0 & 0 & 0 & 0 & \vdots & 0
\end{bmatrix}
\]

(a) Which columns are the pivot columns of the matrix?
(b) What are the free variables?
(c) Express the full solution in parametric form.
(d) Without doing any further calculations, how many solutions does the equation $Ax = 0$ have? Justify your answer.
(e) If you were given the matrix $A$ and the vector $b$ in MATLAB, what further commands would you use to calculate the reduced row-echelon augmented matrix used to find the solution to $Ax = b$.

4. Find an LU factorization of the following matrix:

\[
\begin{bmatrix}
2 & 3 & 2 \\
4 & 13 & 9 \\
-6 & 5 & 4
\end{bmatrix}
\]

5. Consider the linear transformation from $\mathbb{R}^4 \to \mathbb{R}^4$ defined by:

$$T(x_1, x_2, x_3, x_4) = (x_1 + 2x_2, 0, 2x_2 + x_4, x_2 - x_4)$$

(a) What is the matrix that represents this linear transformation?
(b) Is this linear transformation onto? Why or why not?
(c) Is this linear transformation one-to-one? Why or why not?

6. Calculate the determinant of the $4 \times 4$ tridiagonal matrix below using whatever method you find most convenient.

\[
\begin{bmatrix}
2 & 2 & 0 & 0 \\
2 & 2 & 2 & 0 \\
0 & 2 & 2 & 2 \\
0 & 0 & 2 & 2
\end{bmatrix}
\]

From your calculation, do the columns of the matrix span $\mathbb{R}^4$? Why or why not?

### 3 Extra Credit

This will only be worth a few percent of the total points, so make sure you’ve checked your work on the other problems first!

Find formulae for the submatrices $X$, $Y$, and $Z$ in terms of $A$ and $B$. State any assumptions you need to make about the matrices.

\[
\begin{bmatrix}
X & 0 & 0 \\
Y & 0 & I
\end{bmatrix}
\begin{bmatrix}
A & Z \\
0 & 0
\end{bmatrix}
= \begin{bmatrix}
I & 0 \\
0 & I
\end{bmatrix}
\]