Review Questions For Midterm

1. **Complex algebra.** Give all of your answers in rectangular coordinates.
   a. Solve the system
      \[(1 + i)z_1 + 2iz_2 = 2\]
      \[z_1 - (2 - i)z_2 = 3i\]
   b. Solve the quadratic equation
      \[z^2 + 2iz - 10 = 0.\]
   c. Find all the solutions of the equation \(z^4 = 1 + 2i.\) *(Round your solutions to 4 decimal places.)*

2. Use Gauss-Jordan elimination to find the **reduced-row-echelon form** of the matrices \(A\) and \(B,\) below.

\[
A = \begin{bmatrix} 2 & 3 & -1 & 3 \\ 1 & -2 & 3 & 5 \\ 1 & 4 & -3 & -1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 2 & 3 & 7 & 0 & 8 \\ 2 & 4 & 8 & 2 & 4 \\ 2 & 1 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 3 \end{bmatrix}.
\]

3. Write down the systems of equations for which the matrices \(A\) and \(B,\) in problem (2) above, are the **augmented** matrices. Based on your work in the problem (2), determine the solution sets for these two systems. If appropriate, describe the solution sets using one or more parameters.

4. For the matrices \(A\) and \(B\) in problem (2), which of the products \(AB\) and \(BA\) is defined? Compute the one(s) that are.

5. Find the inverses of the matrices \(U, V\) and \(W,\) below.
   \[
   U = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \quad V = \begin{bmatrix} 1 & 1 & 2 \\ 0 & -1 & 3 \\ 2 & 0 & -3 \end{bmatrix} \quad W = \begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 2 & 1 & 1 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 2 \end{bmatrix}.
   \]

6. Mark each of the following statements as either **true** or **false.** Justify each claim of ‘true’ with a brief argument, and justify each claim of ‘false’ by giving an example where the statement fails to be true.
   (i) If \(A\) and \(B\) are both \(n \times n\) matrices and \(AB\) is invertible, then \(A\) and \(B\) are both invertible.
   (ii) Given two matrices \(A\) and \(B,\) such the products \(AB\) and \(BA\) are both defined, then \(AB = BA.\)
   (iii) Every homogeneous system of linear equations is consistent.
   (iv) If the \(5 \times 7\) matrix \(M\) has 5 pivots, then the equation \(M\mathbf{x} = \mathbf{b}\) has a solution for every \(\mathbf{b}\) in \(\mathbb{R}^5.\)
   (v) If the \(5 \times 7\) matrix \(M\) has 5 pivots, and the equation \(M\mathbf{x} = \mathbf{b}\) has a solution, then that solution is unique.
   (vi) If the rank of the \(4 \times 3\) matrix \(A\) is 3, then the system \(A\mathbf{x} = \mathbf{b}\) is consistent for every \(\mathbf{b}\) in \(\mathbb{R}^4.\)
   (vii) If \(A, B\) and \(C\) are all \(2 \times 2\) matrices, \(C\) is not the 0 matrix and \(AC = BC,\) then \(A = B.\)
   (viii) If \(A\) is an \(4 \times 4\) matrix and the system \(A\mathbf{x} = \mathbf{b}\) is consistent for every \(\mathbf{b}\) in \(\mathbb{R}^4,\) then \(A\) is invertible.

7. **Polynomial interpolation.** Find a polynomial of degree 3 (or less) that interpolates the points \((-1, 1), \ (1, 2), \ (2, 4)\) and \((3, 1).\) Is the solution unique? Why?

8. Section 1.3, problems 15, 17, 21; Section 2.3, problem 23; Section 2.4, problems 23, 33, 43.