1. Compute the derivatives, using the rules of differentiation,$^\dagger$ (no limits necessary).

a. \( f(x) = 3x^5 - 2x^4 + 5x^2 - 3x - 2 \), \( f'(x) = \)

b. \( y = \sqrt{x} - \frac{2}{x^3} \), \( \frac{dy}{dx} = \)

c. \( g(t) = \frac{t^2 + 3t - 1}{3t^4 + 5} \), \( g'(t) = \)

d. \( u = \sqrt[3]{v^2 + 3v + 5} \), \( \frac{du}{dv} = \)

e. \( y = (x^2 + x + 1)^3 \), \( y' = \)

f. \( h(x) = \sqrt{2x^2 + 3x - 5} \), \( h'(x) = \)

g. \( l(s) = (s^2 + 2)^{3/2} \cdot (4s + 7) \), \( \frac{dl}{ds} = \)

2. Compute the derivative of \( y = (x^2 + 1)(x^2 - 2x + 2) \) in two different ways, once using the product rule and once without the product rule, and verify that you get the same answer both times.

3. Find the equation of the tangent line to the graph \( y = \sqrt{x} - \frac{1}{\sqrt{x}} \) at the point \((1,0)\).

4. The demand equation for a firm’s product is \( p = 200 - 0.3q^{2/3} \), where \( p \) is the price, measured in $100s, and \( q \) is output (=demand) measured in 1000s of units.

a. Find the firm’s revenue function and the firm’s marginal revenue function. What is the firm’s marginal revenue when \( q = 1000 \)?

b. Use your answer to part a. to estimate the change in the firm’s revenue when output increases from 1,000,000 units to 1,000,150.

Hints/Suggestions:

- Pay attention to the units for \( p, q \) and \( r \), \( (r \) and \( p \) are measured in the same units).

$^\dagger$These are the rules described in sections 11.2 - 11.5 in the textbook.
5. The demand function for a monopolistic firm is \( p = 300 - 0.4q \), and the firm’s cost function is \( c = 0.05q^2 + 30q + 1000 \). Find the level of output, \( q \), for which marginal revenue equals marginal cost.

6. The national savings function for Slugsylvania is

\[
S = \frac{2Y^2 + 5Y + 7}{11Y + 111},
\]

where \( Y \) is annual income and \( S \) is annual savings, and both are measured in $ billions.

a. Compute the marginal propensity to save and the marginal propensity to consume when \( Y = 10 \). Round your answer to the nearest million.

b. Approximately what proportion of income is saved when \( Y \) is very large (as \( Y \to \infty \))?  

7. The production function for ACME Widgets is given by

\[
q = (5l + 4)^{2/3},
\]

where \( q \) is the firm’s weekly output, measured in 1000s of widgets and \( l \) is the firm’s weekly labor input, measured in $1000’s. The firm’s marginal revenue function is given by

\[
\frac{dr}{dq} = 80q^{-1/4},
\]

where revenue and marginal revenue are measured in $1000’s.

a. Compute the firm’s marginal product of labor and marginal revenue product when the firm’s labor input is $12,000 a week.

b. By approximately how many widgets/week will the firm’s output change if they increase the weekly labor input from $12,000 to $12,500?

c. By approximately how much will the firm’s revenue change if they increase the weekly labor input from $12,000 to $12,500?

\[^{\text{‡}}\text{See section 11.3 and SN 5}\]