

Final Exam

Instructions

• There are 6 questions worth a total of 54 points. 100%=50 points.
• No notes or books.
• You may use a simple scientific calculator. No graphing or programmable calculators.
• Take your time. Answer each question completely. Check your answers.
• For full credit, show all your work and reasoning.

Good Luck!!!

NAME:__________________________________________

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1. (8 pts) A firm’s production function is given by

\[ Q = 20K^{3/5}L^{2/5}, \]

where \( Q \) is the firm’s monthly output, \( K \) is the firm’s monthly capital input and \( L \) is the firm’s monthly labor input. Furthermore, each unit of capital input costs the firm $1000, each unit of labor input costs the firm $600 and the their monthly budget for these inputs is $300,000.

Find the levels of capital and labor input that the firm should use to maximize their monthly output, given the costs of the inputs and the budget constraint. What is the firm’s maximum monthly output?

Be sure to explain how you know that your answer produces the absolute maximum monthly output for the firm.
2. The demand equation for a monopolistic firm’s product is given by

\[ p = 54 - 0.4q, \]

where \( p \) is the price per unit that the firm charges (in dollars) and \( q \) is the daily demand for the firm’s product. The firm’s daily cost function is

\[ c = 10q + 80, \]

where \( c \) is also measured in dollars.

\textbf{(a)} (8 pts) Find the price that the firm should set to maximize its daily profit, the profit-maximizing output and the maximum daily profit. Be sure to \textit{explain} how you know that the profit you found is the firm’s absolute maximum.
(b) (2 pts) The local government imposes a $1.60 per unit production tax on the firm. How much of this tax should the firm pass on to consumers (in the form of a price increase) to maximize their profit under the new tax? Show your work.
3. Consider the function \( g(u) = \frac{4u + 3}{u^2 + 1} \).

(a) (8 pts) Find the critical points of \( g(u) \) and classify the critical values as relative minima, relative maxima or neither. You may use either the first derivative test or the second derivative test—your choice.

(b) (2 pts) Does \( g(u) \) attain an **absolute** maximum value on the interval \((0, \infty)\)? Justify your answer.
4. The demand equation for the monopolistic firm ACME Widgets is given by

\[ q = 1.2(600 - 4p)^{3/2}, \]

where \( p \) is the price of a widget and \( q \) is weekly demand for widgets.

(a) (4 pts) Find the demand, \( q \), and price-elasticity of demand for widgets, \( \eta_{q/p} \) when \( p = 125 \).

(b) (2 pts) Use your answer to part a. to find the approximate percentage change in demand if ACME lowers the price of a widget from \( p = 125 \) to \( p = 122.5 \).
(c) (2 pts) What is ACME’s marginal revenue when $p = 125$? Justify your answer briefly.
5. ACME Widgets’ short-term production function is given by

\[ q = 75(5l - 6)^{2/3}, \]

where \( q \) is ACME’s weekly output (which is equal to the weekly demand for their product) and \( l \) is ACME’s labor input, measured in 40-hour work weeks. E.g., if \( l = 10 \), then ACME’s workers are working for a total of \( 10 \times 40 = 400 \) hours a week.

The demand equation for ACME’s product is given in the previous problem.

(a) (4 pts) What is ACME’s output and marginal product, \( dq/dl \), when \( l = 14 \)?

(b) (2 pts) Use your answers to part (a), above, and problem 4(c) to find ACME’s marginal revenue product \( dr/dl \) when \( l = 14 \). (You may assume that the demand for the firm’s product is equal to their output.)
(c) (2 pts) If ACME hires a new employee to work 20 hours a week, what is the approximate effect on their revenue?
6. The consumption function for a small nation is given by

\[ C = \frac{8Y^2 + 25}{9Y + 5}, \]

where \( Y \) is the nation’s annual income and \( C \) is the nation’s annual consumption, both measured in billions of dollars.

(a) (6 pts) What are the nation’s marginal propensities to consume and to save when its annual income is $5 billion?
(b) (2 pts) Use your answer to part a. to estimate the changes in savings and consumption when the nation’s income increases from $5 billion to $5.2 billion.

(c) (2 pts) Compute the limit \( \lim_{Y \to \infty} \frac{dC}{dY} \) and interpret your answer in economic terms.