Final Exam

Instructions

• There are 6 questions worth a total of 54 points. 100%=50 points.

• No notes or books.

• You may use a simple scientific calculator. No graphing or programmable calculators.

• Take your time. Answer each question completely. Check your answers.

• For full credit, show all your work and reasoning.

Good Luck!!!
1. (9 pts) The Smith family’s prandial utility function is given by

\[ U(x, y) = 8 \ln x + 17 \ln y, \]

where \( U \) is the monthly utility they derive from consuming \( x \) meals at home and \( y \) meals in restaurants. The price of a home-cooked meal is \( p_x = \$6.00 \) and the price of a restaurant meal is \( p_y = \$25.00 \).

Find the number of home-cooked meals and restaurant meals that the Jones family should consume each month to maximize their prandial utility if their monthly food budget is \( B = \$750.00 \). What is the maximum utility?

**Explain** how you know that the utility you found is the absolute maximum utility for the given budget.
2. (9 pts) The demand equation for a monopolistic firm’s product is given by

\[ p = 40 - 0.1q^2, \]

where \( p \) is the price per unit that the firm charges (in dollars) and \( q \) is the daily demand for the firm’s product. The firm’s daily cost function is

\[ c = 10q + 80, \]

where \( c \) is also measured in dollars.

Find the price that the firm should set to maximize its daily profit and the maximum profit and explain how you know that the profit you found is the firm’s absolute maximum.
3. Consider the function \( g(u) = 50u^2e^{-0.5u} \).

(a) (6 pts) Find the critical points of \( g(u) \) and classify the critical values as relative minima, relative maxima or neither. You may use either the first derivative test or the second derivative test—your choice.

(b) (2 pts) Does \( g(u) \) attain an absolute maximum value on the interval \((0, \infty)\)? Justify your answer.

(c) (1 pt) Does \( g(u) \) attain an absolute maximum value on the interval \((-\infty, \infty)\)? Justify your answer.
4. The demand equation for the monopolistic firm ACME Widgets is given by
\[ q = 1.2(600 - 4p)^{3/2}, \]
where \( p \) is the price of a widget and \( q \) is monthly demand for widgets.

a. (5 pts) Find the price-elasticity of demand for widgets when \( p = 125 \).

b. (2 pts) Use your answer to part a. to find the approximate percentage change in demand if ACME lowers the price of a widget from \( p = 125 \) to \( p = 120 \).

c. (2 pts) What is ACME’s marginal revenue when \( p = 125 \)? Justify your answer briefly.
5. ACME Widgets’ short-term production function is given by

\[ q = 75(4l - 76)^{2/5}, \]

where \( q \) is ACME’s monthly output (which is equal to the monthly demand for their product) and \( l \) is ACME’s labor input, measured in employee-hours per week (the total number of hours worked by all ACME employees in a week).

The demand equation for ACME’s product is given in the previous problem.

a. (5 pts) What is ACME’s output and marginal product, \( dq/dl \), when \( l = 275 \)?

b. (2 pts) What is ACME’s marginal revenue product \( dr/dl \) when \( l = 200 \)? (See problem 4 for some necessary information.)

c. (2 pts) If ACME hires a new employee to work 10 hours a week, at a total cost (pay + benefits) of $50 an hour, approximately what effect does this have on their profits, assuming that all other production costs remain unchanged?\(^1\)

\(^1\)You may assume that there are 4 work-weeks in a month.
6. The saving function for a small nation is given by

\[ C = \frac{9Y^2 + 5}{10Y + 150}, \]

where \( Y \) is the nation’s annual income and \( C \) is the nation’s annual consumption, both measured in billions of dollars.

a. (5 pts) What are the nation’s marginal propensities to consume and to save when its annual income is $6 billion?

b. (2 pts) Use your answers to part a. to estimate the changes in savings and consumption when the nation’s income increases from $6 billion to $6.3 billion.

c. (2 pts) Compute \( \lim_{Y \to \infty} \frac{dC}{dY} \), and interpret your answer in economic terms.