1. Compute the derivatives of the functions below.

(a) (3 pts) \( g(x) = \frac{x^2 + x + 1}{x^2 + 3} \)

**Quotient rule:**

\[
g'(x) = \frac{(2x + 1)(x^2 + 3) - 2x(x^2 + x + 1)}{(x^2 + 3)^2} = \frac{-x^2 + 4x + 3}{(x^2 + 3)^2}
\]

(b) (3 pts) \( f(t) = 5(t^2 + 3t + 2)^4 \)

**Chain rule:**

\[
f'(t) = 5 \cdot 4(t^2 + 3t + 2)^3(2t + 3) = 20(t^2 + 3t + 2)^3(2t + 3)
\]

2. (4 pts) A monopolistic firm’s demand equation and cost function are given by

\[
q = 70\sqrt{100 - 3p} \quad \text{and} \quad c = 0.05q^2 + 15q + 500.
\]

Find \( \frac{dc}{dp} \) when \( p = 12 \). (\textit{Suggestion:} chain rule)

(i) When \( p = 12 \), \( q = 70\sqrt{100 - 36} = 70 \cdot 8 = 560. \)

(ii) \( \frac{dq}{dp} = 70 \cdot \frac{1}{2} (100 - 3p)^{-1/2}(-3) = -105(100 - 3p)^{-1/2} \), so \( \frac{dq}{dp} \bigg|_{p=12} = -\frac{105}{8} \).

(iii) \( \frac{dc}{dq} = 0.1q + 15 \), so \( \frac{dc}{dq} \bigg|_{q=560} = 56 + 15 = 71. \)

(iv) \( \frac{dc}{dp} \bigg|_{p=12} = \frac{dc}{dq} \bigg|_{q=560} \cdot \frac{dq}{dp} \bigg|_{p=12} = 71 \cdot (-105/8) = -7455/8 = -931.875. \)