1. Compute the indicated derivatives of the functions below.

(a) (3 pts) \( g(x) = 5x^3 + 3x^2 - 7x + 1 \implies g'(x) = 15x^2 + 6x - 7 \) and \( g''(x) = 30x + 6 \)

(b) (3 pts) \( y = \ln(x^2 + 3x + 1) \implies \frac{dy}{dx} = \frac{2x + 3}{x^2 + 3x + 1} \)
\[ \text{and} \quad \frac{d^2y}{dx^2} = \frac{2(x^2 + 3x + 1) - (2x + 3)(2x + 3)}{(x^2 + 3x + 1)^2} = -\frac{2x^2 + 6x + 7}{(x^2 + 3x + 1)^2} \]

2. (4 pts) Find the \textit{quadratic} Taylor polynomial for the function \( f(x) = \sqrt{x} \), centered at \( x_0 = 100 \) and use it to estimate \( \sqrt{102} \).

In general, \( T_2(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{f''(x_0)}{2}(x - x_0)^2 \). In this case, \( x_0 = 100 \) and \( f(x) = x^{1/2} \), so \( f'(x) = \frac{1}{2}x^{-1/2} \) and \( f''(x) = -\frac{1}{4}x^{-3/2} \), and this means that

\[
\begin{align*}
    f(x_0) &= \sqrt{100} = 10, \\
    f'(x_0) &= \frac{1}{2}100^{-1/2} = \frac{1}{20}, \\
    f''(x_0) &= -\frac{1}{4}100^{-3/2} = -\frac{1}{4000}.
\end{align*}
\]

Therefore, the quadratic Taylor polynomial for \( f(x) = \sqrt{x} \) centered at \( x_0 = 100 \) is

\[
T_2(x) = 10 + \frac{1}{20}(x - 100) - \frac{1}{8000}(x - 100)^2,
\]
and

\[
\sqrt{102} \approx T_2(102) = 10 + \frac{2}{20} - \frac{4}{8000} = 10.0995.
\]