1. (10 pts) The demand equation for a monopolistic firm’s product is given by \( p = 120 - 0.4q \), where \( p \) is the price per unit (in dollars) of the firm’s good and \( q \) is the weekly demand for this good (measured in individual units). The firm’s cost function is \( c = 0.1q^2 + 15q + 1000 \), where \( c \) is the weekly cost (in dollars) of producing \( q \) units, and where the firm’s weekly output is equal to the weekly demand.

Find the price that the firm should set to *maximize its profit*, the profit maximizing output and the maximum profit itself. You must verify that the critical profit that you found is indeed the firm’s maximum profit. *Show your work.*

(1) Revenue function: \( r = pq = 120q - 0.4q^2 \)

(2) Profit function: \( \Pi = r - c = 120q - 0.4q^2 - (0.1q^2 + 15q + 1000) = -0.5q^2 + 105q - 1000. \)

(3) Critical output: \( \Pi' = -q + 105 = 0 \implies q^* = 105. \)

(4) Critical price: \( p^* = 120 - 0.4q^* = 120 - 42 = 78. \)

(5) Critical profit: \( \Pi^* = -0.5(q^*)^2 + 105q^* - 1000 = 4512.50 \)

(6) Verification that \( \Pi^* \) is the max profit: \( \Pi'' = -1 < 0 \), so \( \Pi^* \) is a relative maximum, and since there is only one critical point, \( \Pi^* \) is the absolute maximum profit.