Final Exam

Instructions

• Please turn off all phones and other electronic devices.

• There are 6 questions worth a total of 54 points. 100% = 50 points.

• No notes or books. A table of integration formulas is provided.

• You may use a simple scientific calculator. No graphing or programmable calculators.

• Read the questions carefully and check your answers.

• For full credit—show all your work.

Good Luck!!!

NAME: ____________________________________________

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Selected Integration Formulas

Basic rules.

1. $\int u^k \, du = \frac{u^{k+1}}{k+1} + C$, $k \neq -1$.

2. $\int \frac{1}{u} \, du = \ln |u| + C$.

3. $\int e^u \, du = e^u + C$.

4. $\int f(u) \pm g(u) \, du = \int f(u) \, du \pm \int g(u) \, du$.

5. $\int c \cdot f(u) \, du = c \cdot \int f(u) \, du$.

Rational forms containing $(a + bu)$.

6. $\int \frac{du}{a + bu} = \frac{1}{b} \ln |a + bu| + C$.

7. $\int \frac{udu}{a + bu} = \frac{u}{b} - \frac{a}{b^2} \ln |a + bu| + C$.

8. $\int \frac{u^2 \, du}{a + bu} = \frac{u^2}{2b} - \frac{au}{b^2} + \frac{a^2}{b^3} \ln |a + bu| + C$.

9. $\int \frac{u^2 \, du}{(a + bu)^2} = \frac{u}{b^2} - \frac{a^2}{b^3(a + bu)} - \frac{2a}{b^3} \ln |a + bu| + C$.

Forms containing $\sqrt{a + bu}$.

10. $\int u \sqrt{a + bu} \, du = \frac{2(3bu - 2a)(a + bu)^{3/2}}{15b^2} + C$.

11. $\int \frac{u \, du}{\sqrt{a + bu}} = \frac{2(bu - 2a)\sqrt{a + bu}}{3b^2} + C$.

12. $\int \frac{u^2 \, du}{\sqrt{a + bu}} = \frac{2(3b^2u^2 - 4abu + 8a^2)\sqrt{a + bu}}{15b^3} + C$.

Exponential and logarithmic forms.

13. $\int e^{au} \, du = \frac{e^{au}}{a} + C$.

14. $\int u e^{au} \, du = \frac{e^{au}}{a^2} (au - 1) + C$.

15. $\int u^n e^{au} \, du = \frac{u^n e^{au}}{a} - \frac{n}{a} \int u^{n-1} e^{au} \, du$.

16. $\int u^n \ln u \, du = \frac{u^{n+1} \ln u}{n+1} - \frac{u^{n+1}}{(n+1)^2} + C$, $n \neq -1$. 
1. (8 pts) Compute the present value of a continuous annuity that pays at the annual rate \( f(t) = 500t \) for \( T = 20 \) years, assuming that interest is compounded continuously at the rate \( r = 4.5\% \).
2. (8 pts) Find the Consumers’ surplus and Producers’ surplus at equilibrium for the market whose supply and demand equations are given below.

- **Supply**: \( p = 15 + 0.1q \).
- **Demand**: \( p = 120 - 0.04q^2 \).
3. The average monthly demand \((Q)\) for a ACME Widgets’ product is related to the price of their Widgets \((p)\), the average price of substitutes for Widgets \((p_s)\) and the average monthly household income in the market for the firm’s product \((Y)\), by the equation

\[
Q = \frac{80(2Y + 16p_s - 1600)^{3/4}}{3p + 10},
\]

where \(Q\) is measured in 1000s of Widgets, and the prices and income are all measured in dollars.

a. (6 pts) Compute \(Q_p\), \(Q_{p_s}\), \(Q_Y\) when \(p = 30\), \(p_s = 31\) and \(Y = 2600\).

b. (2 pts) Compute the income-elasticity of demand when \(p = 30\), \(p_s = 31\) and \(Y = 2600\).

c. (2 pts) Suppose that income remains fixed and both prices increase by $1. Use your answer to a. to estimate the change in demand for ACME’s product.

*Round your answers to 2 decimal places.*
4. (8 pts) Find the critical points of the function

\[ f(x, y) = x^3 + x^2 + 4xy + y^2 + 12y + 3 \]

and classify the critical values using the second derivative test.
5. ACME Widget’s (AW) production function is given by

\[ Q = 25K^{3/5}L^{2/5}, \]

where \( Q \) is the firm’s annual output, measured in widgets, \( K \) is the firm’s monthly capital input and \( L \) is the firm’s monthly labor input. The price per unit of capital is \( p_K = $10,000 \) and the price per unit of labor is \( p_L = $4,000 \).

a. (6 pts) Find the levels of capital and labor input that AW should use to minimize the cost of producing \( Q_0 = 25,000 \) widgets. What is the minimum cost?
b. (2 pts) What is AW’s marginal cost at that level of production? Explain your answer.

c. (2 pts) Use the envelope theorem and linear approximation to estimate the change in AW’s (minimum) cost of producing 25,000 widgets, if the price per unit of labor increases to $4,100 (assuming that all else stays the same).
6. The Jones family’s utility function is given by

\[ U(x, y, z) = 10 \ln x + 6 \ln y + 9 \ln z, \]

where \( x, y \) and \( z \) are the quantities of X-goods, Y-goods and Z-goods that they consume per month. The average prices of these goods are \( p_x = $12, p_y = $5 \) and \( p_z = $10 \), respectively.

a. (8 pts) Find the quantities of X-goods, Y-goods and Z-goods that the Jones family should consume each month to maximize their utility, given that their monthly XYZ-budget is \( B = $3000 \). What is their maximum utility?
b. (2 pts) Use the *envelope theorem* and *linear approximation* to estimate the change in the Jones’ monthly utility if the price of Y-goods *increases* to $5.50 (assuming that all else stays the same).