1. (10 pts) Find the Consumers’ and Producers’ surplus at equilibrium for the market with the supply and demand equations below.

Supply: \( p = 2 + 0.03q^2 \)

Demand: \( p = 70 - 0.5q \)

(a) Find the point of market equilibrium by solving the equation ‘Supply=Demand’:

\[
2 + 0.03q^2 = 70 - 0.5q \implies 0.03q^2 + 0.5q - 68 = 0 \implies q = \frac{-0.5 \pm \sqrt{0.25 + 8.16}}{0.06}
\]

The two roots are \( q_1 = \frac{-0.5 + \sqrt{8.41}}{0.06} = 40 \) and \( q_2 = \frac{-0.5 - \sqrt{8.41}}{0.06} = -\frac{170}{3} \), so the equilibrium demand is \( q^* = q_1 = 40 \) (demand must be positive), and the equilibrium price is \( p^* = 70 - 20 = 50 \).

(b) Consumers’ surplus = \( \int_0^{q^*} (\text{demand} - p^*) \, dq = \int_0^{40} 70 - 0.5q - 50 \, dq \)

\[
= \left[ 20q - \frac{q^2}{4} \right]_0^{40} = (800 - 400) - 0 = 400.
\]

(c) Producers’ surplus = \( \int_0^{q^*} p^* - (\text{supply}) \, dq = \int_0^{40} 50 - (2 + 0.03q^2) \, dq \)

\[
= \int_0^{40} 48 - 0.03q^2 \, dq = 48q - 0.01q^3 \bigg|_0^{40} = (1920 - 640) - 0 = 1280.
\]