Final Exam – Make Up

Instructions

• There are 6 questions worth a total of 65 points. 100% = 60 points.

• No notes or books. A table of integration formulas is provided.

• You may use a simple scientific calculator. No graphing or programmable calculators.

• Take your time. Answer each question completely. Check your answers.

• For full credit—explain/show your work.

Good Luck!!!
Selected Integration Formulas

Basic rules.

1. \( \int u^k \, du = \frac{u^{k+1}}{k+1} + C, \quad k \neq -1. \)

2. \( \int \frac{1}{u} \, du = \ln |u| + C. \)

3. \( \int e^u \, du = e^u + C. \)

4. \( \int f(u) \pm g(u) \, du = \int f(u) \, du \pm \int g(u) \, du. \)

5. \( \int c \cdot f(u) \, du = c \cdot \int f(u) \, du. \)

Rational forms containing \((a + bu)\).

6. \( \int \frac{du}{a + bu} = \frac{1}{b} \ln |a + bu| + C. \)

7. \( \int \frac{udu}{a + bu} = \frac{u}{b} - \frac{a}{b^2} \ln |a + bu| + C. \)

8. \( \int \frac{u^2 \, du}{a + bu} = \frac{u^2}{2b} - \frac{au}{b^2} + \frac{a^2}{b^3} \ln |a + bu| + C. \)

9. \( \int \frac{u^2 \, du}{(a + bu)^2} = \frac{u}{b^2} - \frac{a^2}{b^3(a + bu)} - \frac{2a}{b^3} \ln |a + bu| + C. \)

Forms containing \(\sqrt{a + bu}\).

10. \( \int u \sqrt{a + bu} \, du = \frac{2(3bu - 2a)(a + bu)^{3/2}}{15b^2} + C. \)

11. \( \int \frac{u \, du}{\sqrt{a + bu}} = \frac{2(bu - 2a) \sqrt{a + bu}}{3b^2} + C. \)

12. \( \int \frac{u^2 \, du}{\sqrt{a + bu}} = \frac{2(3b^2u^2 - 4abu + 8a^2) \sqrt{a + bu}}{15b^3} + C. \)

Exponential and logarithmic forms.

13. \( \int e^{au} \, du = \frac{e^{au}}{a} + C. \)

14. \( \int ue^{au} \, du = \frac{e^{au}}{a^2} (au - 1) + C. \)

15. \( \int u^n e^{au} \, du = \frac{u^n e^{au}}{a} - \frac{n}{a} \int u^{n-1} e^{au} \, du. \)

16. \( \int u^n \ln u \, du = \frac{u^{n+1} \ln u}{n+1} - \frac{u^{n+1}}{(n+1)^2} + C, \quad n \neq -1. \)
1. (10 pts) Compute the *present value* of a continuous annuity that pays at the annual rate $f(t) = 2000t$ for $T = 10$ years, assuming that interest is compounded continuously at the rate $r = 4.2\%$.

b. (5 pts) Compute the *Gini coefficient* (of inequality) for the nation whose income distribution function is given by $f(x) = 0.7x^2 + 0.3x$. 
2. (10 pts) Find the Consumers’ surplus and Producers’ surplus at equilibrium for the market whose supply and demand equations are given below.

- Supply: \( p = 15 + 0.75q \),
- Demand: \( p = 50 - 0.05q^2 \).
3. The Jones family’s utility function is given by

\[ U(x, y, z) = 8 \ln x + 5 \ln y + 7 \ln z, \]

where \( x, y \) and \( z \) are the quantities of Xidgets, Yidgets and Zidgets, respectively, that they consume per month, for which the average prices per unit are \( p_x = $10, \ p_y = $8 \) and \( p_z = $12 \), respectively.

a. (6 pts) Find the quantities of Xidgets, Yidgets and Zidgets that the Jones family should consume each month to maximize their utility, given that their monthly XYZ-budget is \( B = $4800 \).

b. (2 pts) By approximately how much will the Jones' have to increase their monthly XYZ-budget from its current level to increase their (maximum) utility by 2 utils?

c. (2 pts) By approximately how much will the Jones’ (maximum) utility change from the value you found in a., if the average price of a Xidget increases by $1, assuming that the other prices and their budget stay the same? Justify your answer in terms of the envelope theorem.
4. (10 pts) Find the critical points of the function

\[ f(x, y) = x^3 + 2x^2 + 4xy + y^2 + 2y + 3 \]

and classify the critical values using the second derivative test.
5. The average monthly demand ($Q$) for a monopolistic firm’s product is related to the price of their product ($P$), the average price of substitutes for their product ($P_s$) and the average monthly household income in the market for the firm’s product ($Y$), by the equation:

$$Q = \frac{150(2Y + 16P_s - 1600)^{3/4}}{3P + 10}.$$

a. (6 pts) Compute $Q$, $Q_p$, $Q_{P_s}$, and $Q_Y$ when $P = 30$, $P_s = 31$ and $Y = 2600$.

b. (2 pts) What is the income-elasticity of demand at the point in part a.?

c. (2 pts) Suppose that income stays fixed, but both prices decrease by $1$. Use your answer to a. to estimate the change in demand for the firm’s product.
6. (10 pts) The price elasticity of demand, $\eta_{q/p}$, for a monopolistic firm’s product is proportional to

$$\frac{p^{1/3}}{q},$$

where $p$ is the price of the firm’s good and $q$ is the demand for their product. Find the demand function $q = f(p)$ for the firm’s product, given that $f(10) = 100$ and $f(20) = 80$. 