Final Exam

Instructions

• Please turn off all phones and other electronic devices.
• There are 7 questions worth a total of 44 points. 100% = 40 points.
• No notes or books. A table of integration formulas is provided.
• You may use a simple scientific calculator. No graphing or programmable calculators.
• Read the questions carefully and check your answers.
• For full credit—show all your work.

Good Luck!!!

NAME: __________________________

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Selected Integration Formulas

Basic rules.

1. \( \int u^k \, du = \frac{u^{k+1}}{k+1} + C, \quad k \neq -1. \)
2. \( \int \frac{1}{u} \, du = \ln |u| + C. \)
3. \( \int e^u \, du = e^u + C. \)
4. \( \int f(u) \pm g(u) \, du = \int f(u) \, du \pm \int g(u) \, du. \)
5. \( \int c \cdot f(u) \, du = c \cdot \int f(u) \, du. \)

Rational forms containing \((a + bu)\).

6. \( \int \frac{du}{a + bu} = \frac{1}{b} \ln |a + bu| + C. \)
7. \( \int \frac{u \, du}{a + bu} = \frac{u}{b} - \frac{a}{b^2} \ln |a + bu| + C. \)
8. \( \int \frac{u^2 \, du}{a + bu} = \frac{u^2}{2b} - \frac{au}{b^2} + \frac{a^2}{b^3} \ln |a + bu| + C. \)
9. \( \int \frac{u^2 \, du}{(a + bu)^2} = \frac{u}{b^2} - \frac{a^2}{b^3(a + bu)} - \frac{2a}{b^3} \ln |a + bu| + C. \)

Forms containing \(\sqrt{a + bu}\).

10. \( \int u \sqrt{a + bu} \, du = \frac{2(3bu - 2a)(a + bu)^{3/2}}{15b^2} + C. \)
11. \( \int \frac{u \, du}{\sqrt{a + bu}} = \frac{2(bu - 2a)\sqrt{a + bu}}{3b^2} + C. \)
12. \( \int \frac{u^2 \, du}{\sqrt{a + bu}} = \frac{2(3b^2u^2 - 4abu + 8a^2)\sqrt{a + bu}}{15b^3} + C. \)

Exponential and logarithmic forms.

13. \( \int e^{au} \, du = \frac{e^{au}}{a} + C. \)
14. \( \int ue^{au} \, du = \frac{e^{au}}{a^2} (au - 1) + C. \)
15. \( \int u^n e^{au} \, du = \frac{u^n e^{au}}{a} - \frac{n}{a} \int u^{n-1} e^{au} \, du. \)
16. \( \int u^n \ln u \, du = \frac{u^{n+1} \ln u}{n+1} - \frac{u^{n+1}}{(n+1)^2} + C, \quad n \neq -1. \)
1. (6 pts) Compute the present value of a continuous annuity that pays at the annual rate $f(t) = 1500t$ for $T = 10$ years, assuming that interest is compounded continuously at the rate $r = 4.2\%$. 
2. (6 pts) Find the Consumers’ surplus and Producers’ surplus at equilibrium for the market whose supply and demand equations are given below.

- **Supply**:  \( p = 10 + 0.1q^2 \),
- **Demand**:  \( p = 60 - 0.5q \).
3. (6 pts) A firm’s marginal cost function is given by

\[ \frac{dc}{dq} = 20q\sqrt{25 + 5q}. \]

By how much does the firm’s cost change if they increase their output from \( q_1 = 15 \) to \( q_2 = 40 \)?
4. (6 pts) The growth in the population of a small island is modeled by the differential equation

\[ \frac{dy}{dt} = k y^{3/4}, \]

where \( y(t) \) is the number of people on the island at time \( t \) (\( t \) measured in years) and \( k \) is an unknown constant of proportionality.

In the year 2000 the population was 1200 and in 2010 the population was 1400. What will the island’s population be in 2020, rounded to the nearest whole number?
5. (6 pts) Find the critical points of the function

\[ f(x, y) = \frac{1}{3}x^3 + 4x^2 - 2xy + y^2 + 8x + 7 \]

and classify the critical values using the second derivative test.
6. The average monthly demand \((Q)\) for a monopolistic firm’s product is related to the price of their product \((p)\), the average price of substitutes for their product \((p_s)\) and the average monthly household income in the market for the firm’s product \((Y)\), by the equation:

\[
Q = \frac{100(3Y + 24p_s + 840)^{4/5}}{3p + 5}.
\]

a. (4 pts) Compute \(Q_p\) and \(Q_{p_s}\) when \(p = 15\), \(p_s = 14\) and \(Y = 2200\).

b. (2 pts) Suppose that income stays fixed, but \(p\) increases by \$0.50 and \(p_s\) increases by \$1.50. Use your answer to a. to estimate the change in monthly demand for the firm’s product.

*Round your answers to 2 decimal places.*
7. The Smith family’s utility function is given by

\[ F(u, v, w) = 10 \ln u + 8 \ln v + 7 \ln w, \]

where \( u, v \) and \( w \) are the quantities of Udgets, Vidgets and Widgets that they consume per month, respectively. The average prices per unit of Udgets, Vidgets and Widgets are \( p_u = $20 \), \( p_y = $15 \) and \( p_z = $8 \), respectively.

a. (6 pts) Find the quantities of Udgets, Vidgets and Widgets that the Smith family should consume each month to maximize their utility, given that their monthly UVW-budget is \( B = $6000 \).

b. (2 pts) By approximately how much will the Smith’s (maximum) utility change from the value you found in a., if the average price of a Widget increases by $2, assuming that the other prices and their budget stay the same? Justify your answer in terms of the envelope theorem.