# Final Exam

**Instructions**

- Please turn off all phones and other electronic devices.
- There are 6 questions worth a total of 54 points. 100% = 50 points.
- No notes or books. A table of integration formulas is provided.
- You *may* use a simple scientific calculator. *No* graphing or programmable calculators.
- *Read the questions carefully and check your answers.*
- *For full credit—show all your work.*

*Good Luck!!!*

**NAME:** __________________________________________

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Selected Integration Formulas

Basic rules.

1. \[ \int u^k \, du = \frac{u^{k+1}}{k+1} + C, \quad k \neq -1. \]

2. \[ \int \frac{1}{u} \, du = \ln |u| + C. \]

3. \[ \int e^u \, du = e^u + C. \]

4. \[ \int f(u) \pm g(u) \, du = \int f(u) \, du \pm \int g(u) \, du. \]

5. \[ \int c \cdot f(u) \, du = c \cdot \int f(u) \, du. \]

Rational forms containing \((a + bu)\).

6. \[ \int \frac{du}{a + bu} = \frac{1}{b} \ln |a + bu| + C. \]

7. \[ \int \frac{u \, du}{a + bu} = \frac{u}{b} - \frac{a}{b^2} \ln |a + bu| + C. \]

8. \[ \int \frac{u^2 \, du}{a + bu} = \frac{u^2}{2b} - \frac{au}{b^2} + \frac{a^2}{b^3} \ln |a + bu| + C. \]

9. \[ \int \frac{u^2 \, du}{(a + bu)^2} = \frac{u}{b^2} - \frac{a^2}{b^3(a + bu)} - \frac{2a}{b^3} \ln |a + bu| + C. \]

Forms containing \(\sqrt{a + bu}\).

10. \[ \int u \sqrt{a + bu} \, du = \frac{2(3bu - 2a)(a + bu)^{3/2}}{15b^2} + C. \]

11. \[ \int \frac{u \, du}{\sqrt{a + bu}} = \frac{2(bu - 2a)\sqrt{a + bu}}{3b^2} + C. \]

12. \[ \int \frac{u^2 \, du}{\sqrt{a + bu}} = \frac{2(3b^2u^2 - 4abu + 8a^2)\sqrt{a + bu}}{15b^3} + C. \]

Exponential and logarithmic forms.

13. \[ \int e^{au} \, du = \frac{e^{au}}{a} + C. \]

14. \[ \int ue^{au} \, du = \frac{e^{au}}{a^2} (au - 1) + C. \]

15. \[ \int u^n e^{au} \, du = \frac{u^n e^{au}}{a} - \frac{n}{a} \int u^{n-1} e^{au} \, du. \]

16. \[ \int u^n \ln u \, du = \frac{u^{n+1} \ln u}{n + 1} - \frac{u^{n+1}}{(n + 1)^2} + C, \quad n \neq -1. \]
1. (8 pts) Compute the present value of a continuous annuity that pays at the annual rate \( f(t) = 1000t \) for \( T = 10 \) years, assuming that interest is compounded continuously at the rate \( r = 5.5\% \).
2. (8 pts) Find the Consumers’ surplus and Producers’ surplus at equilibrium for the market whose supply and demand equations are given below.

- **Supply**: \( p = 5 + 0.125q \),
- **Demand**: \( p = 90 - 0.05q^2 \).
3. The average monthly demand \((Q)\) for a ACME Widgets’ product is related to the price of their Widgets \((p)\), the average price of substitutes for Widgets \((p_s)\) and the average monthly household income in the market for the firm’s product \((Y)\), by the equation

\[
Q = \frac{100(2Y + 15p_s - 1650)^{3/5}}{2p + 10},
\]

where \(Q\) is measured in 1000s of Widgets, and the prices and income are all measured in dollars.

a. (6 pts) Compute \(Q\), \(Q_p\), \(Q_{p_s}\) \(Q_Y\) when \(p = 20\), \(p_s = 25\) and \(Y = 2200\).

b. (2 pts) Compute the income-elasticity of demand when \(p = 20\), \(p_s = 25\) and \(Y = 2200\).

c. (2 pts) Suppose that income remains fixed and both prices increase by $1. Use your answer to a. to estimate the change in demand for ACME’s product.

Round your answers to 2 decimal places.
4. (8 pts) Find the critical points of the function

\[ f(x, y) = x^3 + 2x^2 + 2xy - y^2 - 9y + 1 \]

and classify the critical values using the second derivative test.
5. Industrial Gadget’s (IG) production function is given by

\[ Q = 30K^{2/3}L^{1/3}, \]

where \( Q \) is the firm’s annual output, measured in gadgets, \( K \) is the firm’s monthly capital input and \( L \) is the firm’s monthly labor input. The price per unit of capital is \( p_K = \$5,000 \) and the price per unit of labor is \( p_L = \$3,000 \).

a. (6 pts) Find the levels of capital and labor input that IG should use to minimize the cost of producing \( Q_0 = 10,000 \) gadgets. What is the minimum cost?
b. (2 pts) What is IG's marginal cost at that level of production? Explain your answer.

c. (2 pts) Use the envelope theorem and linear approximation to estimate the change in IG's (minimum) cost of producing 10,000 gadgets, if the price per unit of capital increases to $5,100 (assuming that all else stays the same).
6. The Smith family’s utility function is given by

\[ U(x, y, z) = 7 \ln x + 8 \ln y + 10 \ln z, \]

where \( x, y \) and \( z \) are the quantities of X-wares, Y-wares and Z-wares that they consume per month. The average prices of these wares are \( p_x = $10 \), \( p_y = $15 \) and \( p_z = $20 \), respectively.

a. (8 pts) Find the quantities of X-wares, Y-wares and Z-wares that the Smith family should consume each month to maximize their utility, given that their monthly XYZ-budget is \( B = $4500 \). What is their maximum utility?
b. (2 pts) Use the *envelope theorem* and *linear approximation* to estimate the change in the Smith’s monthly utility if the price of Z-wares *decreases* to $19.50 (assuming that all else stays the same).