1. (10 pts) Find the maximum value of the function

\[ U(x, y, z) = 4 \ln x + 5 \ln y + 6 \ln z \]

subject to the constraint

\[ 5x + 8y + 12z = 3000. \]

(You may assume that the critical value you find is the required maximum — no need for a second-derivative test here.)

1. Lagrangian: \( F(x, y, z, \lambda) = 4 \ln x + 5 \ln y + 6 \ln z - \lambda(5x + 8y + 12z - 3000) \).

2. First order conditions:

\[
\begin{align*}
F_x &= 0 \implies \frac{4}{x} - 5\lambda = 0 \quad \implies \lambda = \frac{4}{5x} \\
F_y &= 0 \implies \frac{5}{y} - 8\lambda = 0 \quad \implies \lambda = \frac{5}{8y} \\
F_z &= 0 \implies \frac{6}{z} - 12\lambda = 0 \quad \implies \lambda = \frac{1}{2z} \\
F_\lambda &= 0 \implies -(5x + 8y + 12z - 3000) = 0 \quad \implies 5x + 8y + 12z = 3000
\end{align*}
\]

3. Finding the critical \( x, y \) and \( z \) values:

\[
(\lambda =) \quad \frac{4}{5x} = \frac{5}{8y} = \frac{1}{2z}.
\]
Comparing the $x$ term to the $y$ term gives

$$32y = 25x \implies y = \frac{25}{32}x,$$

and comparing the $x$ term to the $z$ term gives

$$8z = 5x \implies z = \frac{5}{8}x.$$

Now we substitute these expressions into the constraint, and solve for $x$:

$$5x + 8\left(\frac{25}{32}x\right) + 12\left(\frac{5}{8}x\right) = 3000 \implies (20 + 25 + 30)x = 12000 \implies x^* = 160.$$

Therefore $y^* = \frac{25}{32}x^* = 125$ and $z^* = \frac{5}{8}x^* = 100$.

4. The maximum value: \[ U^* = U(x^*, y^*, z^*) = 4 \ln 160 + 5 \ln 125 + 6 \ln 100 \approx 72.0733. \]