1. (10 pts) Use the method of Lagrange multipliers to find the maximum value of the function
\[ f(x, y) = 10x^{3/4}y^{1/2} \]
subject to the constraint
\[ 9x + 4y = 180. \]
You may assume that the critical value you find is the maximum value.

(1) Lagrangian: \( F(x, y, \lambda) = 10x^{3/4}y^{1/2} - \lambda(9x + 4y - 180). \)

(2) First order equations:
\[ F_x = 7.5x^{-1/4}y^{1/2} - 9\lambda = 0 \]
\[ F_y = 5x^{3/4}y^{-1/2} - 4\lambda = 0 \]
\[ F_\lambda = -(9x + 4y - 180) = 0 \]

(3) Critical point: \( F_x = 0 \implies \lambda = \frac{5y^{1/2}}{6x^{1/4}} \) and \( F_y = 0 \implies \lambda = \frac{5x^{3/4}}{4y^{1/2}}. \)

\[ \lambda = \lambda \implies \frac{5y^{1/2}}{6x^{1/4}} = \frac{5x^{3/4}}{4y^{1/2}} \implies 20y = 30x \implies y = \frac{3}{2}x \]
Substitute into the constraint:

\[ 9x + 4y = 180 \implies 9x + 4 \left( \frac{3}{2}x \right) = 180 \implies 15x = 180 \implies x^* = 12. \]

(4) Conclusion: \( x^* = 12, y^* = \frac{3x^*}{2} = 18 \) and

\[ f^* = f(x^*, y^*) = 10(12)^{3/4}(18)^{1/2} \approx 273.54 \]

is the required maximum value.