Supplementary Problems

1.72 p53
12, 16, 4, 8

2.5 p90. Continue part 1b
4, 6, 13, 16, 20, 23, 28. (38)

3.11 p202
Reus + Disturbance
1, 4, 9

4.9 p272
Expected
49, 47, 1, 11
4, 2, 3, 6, 9
4, 3, 5, 6
\[ \text{Lumination of sum} \]

5.11 p345
Special Distribution
5 (Poisson)

\[ \text{sum of 2 Poissons is Poisson} \]
11 (Geometric)

12
13
18
20
\[
\begin{align*}
\text{(a3)} & \quad \binom{12}{12} \binom{250}{100} \binom{18}{12} \\
& \quad \binom{350}{30} \\
\end{align*}
\]

64) \( \binom{20}{10} \) ways of choosing 10 cards.

\( \binom{4}{2} \) ways of choosing each of 1, 2, 3, 4, 5.

\[
\frac{\binom{4}{5}}{20} = \frac{\binom{20}{10}}{20}
\]

66) \( \binom{r+m}{r} \) ways of placing red balls.

One way that they're in 1st r positions \( = \frac{1}{\binom{r+m}{r}} \).

68) \( \binom{r+1}{r} \) ways of placing red balls can be in 1st \( r+1 \) slots.

68) \( \binom{10}{7} \) ways of choosing envelopes for the red cards.

\( \binom{7}{2} \binom{7-j}{j-3} \) ways of choosing exactly \( j \) red envelopes and \( 7-j \) green envelopes.

\[
\begin{align*}
\Rightarrow & \quad \frac{\binom{7}{j}}{\binom{7-j}{j-3}} \\
& \quad \binom{10}{j} \\
\end{align*}
\]

However, if \( j \) red envelopes contain red cards

Then \( j-4 \) green envelopes contain green cards.

\[
\Rightarrow & \quad \text{the pt is } k = j + j-4 = 2j-4 \text{ matches.}
\]
\[ P(A^c \cup B^c) \]
\[ P(A \cap B) = \frac{P(A \cap B)}{P(B)} \]
\[ P(A \mid B) = \frac{P(A \cap B)}{P(B)} \]
\[ \frac{P(A \cap B)}{\frac{1}{3}} + \frac{P(A \cap B)}{\frac{1}{2}} = \frac{2}{3} \]
\[ \Rightarrow P(A \cap B) = \frac{1}{2} \]
\[ P(A^c \cup B^c) = 1 - \frac{1}{2} = \frac{1}{2} \]

Q5
\[ P(\text{exactly 3} \mid \text{exactly 3} \text{ out of } 10 \text{ are } 6) \]
\[ = \frac{\left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^{10-3}}{\binom{10}{3} \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^{10-3}} = \frac{1}{\binom{10}{3}} \]

Q13
a) \[ P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) \]
\[ + P(A \cap B \cap C) \]
\[ = 0.3 + 0.5 + 0.8 - \left[ 0.3 \times 0.5 + 0.3 \times 0.8 + 0.5 \times 0.8 \right] + \]
\[ 0.3 \times 0.5 \times 0.8 \]
\[ = 0.93 \]

b) \[ P(A \cap B \cap C) + P(A^c \cap B \cap C) + P(A \cap B^c \cap C) \]
\[ = 0.38 \]

Q16
5 balls \( \Rightarrow \) a box,
\[ P(\text{no box contains } 2 \text{ balls}) \]
\[ A_3 \quad \text{event that box i has at least 3 balls,} \]
\[ P(A_i) = \sum_{j=3}^{5} \frac{\binom{5}{j} \left(\frac{1}{2}\right)^{j} \left(\frac{1}{2}\right)^{5-j}}{\binom{5}{j} \left(\frac{1}{2}\right)^{5-j}} = p \]
\[ A_i's \quad \text{are disjoint} \quad \Rightarrow P(\text{at least one of } A_i \text{'s}) = np \]
\[ \Rightarrow P(\text{no box contains } 2 \text{ balls}) = 1 - np \]
E = event that B wins.

1st form: B wins if A misses + B wins, \( P(e) = \frac{5}{6} \times \frac{1}{6} \).

2nd form: both miss on 1st form, + subsequently B wins.

\[ P(e) = \sum_{n=0}^{\infty} \left( \frac{5}{6} \right)^n \left( \frac{1}{6} \right)^n \]

\[ p(e) = \frac{5}{6} \left( \frac{1}{6} \right) + \left( \frac{5}{6} \right)^2 \left( \frac{1}{6} \right)^2 + \left( \frac{5}{6} \right)^3 \left( \frac{1}{6} \right)^3 + \cdots \]

A = head always lands up
down

\[ P(A | B) = \frac{0.8 \times 0.1}{0.8 \times 0.1 + 0.15 \times 0.9} = 0.372. \]

Q24. a) \( P(x = n+1 | x \geq n-2) = \frac{P(x = n+1)}{P(x \geq n-2)} \)

\[ = \frac{n}{(n-1) \left( \frac{1}{2} \right)^n}. \]

\[ \left[ \left( \frac{n}{(n-1)} \right)^n + \left( \frac{n}{(n-1)} \right)^n + \left( \frac{n}{n} \right)^n \right] \left( \frac{1}{2} \right)^n \]

\[ = \frac{n}{\frac{q(n-1)}{2} + n + 1} = \frac{2n}{n^2 + n + 2}. \]

b) \( P(\text{exactly 1 head in 2 tosses}) = \frac{1}{2} \).
3.11 \[ F(z) = P(z \leq z) \]

\[ = P(z \geq x)P(x \leq z) + P(z = y)P(y \leq z) \]

\[ = \frac{1}{2} P(x \leq z) + \frac{1}{2} P(y \leq z) \]

(approximately).

Q.4.

\[ f(x) = \frac{1}{2} e^{-|x|}, \quad -\infty < x < \infty. \]

Area to left of \( x = 0 = \frac{1}{2} \) by symmetry.

\[ \sqrt{2} \pi \int_0^{\infty} \frac{1}{2} e^{-x} \, dx = 0.4 \]

\[ \left[-\frac{1}{2} e^{-x}\right]_0^x = \frac{1}{2} \left(1 - e^{-x_0}\right) \]

\[ \frac{1}{2} \left(1 - e^{-x_0}\right) = 0.4, \quad 1 - e^{-x_0} = 0.8, \quad e^{-x_0} = 0.2, \quad x_0 = \log(0.2) \]

Q.9. A box holds 4 balls of all 3 types. \( \leq \) indicator \( R_U \).

\[ P(A|X=x) = x^2 \]

\[ P(A) = \int_0^1 x^2 f(x) \, dx = \frac{1}{10} \leq P(A) = E[\text{indicator } R_U \text{ for } A] \]
4.1 \( \mathcal{E}(\frac{1}{x}) = \int_0^1 \frac{1}{x} \, dx = \left[ \log x \right]_0^1 = \log 1 - \log 0 \) - this integral does not exist.

4.2 \( \mathcal{E}[x^2] = \sum x^2 \varphi(x) = \frac{1}{n} \sum_{j=1}^n x^2 = \frac{n (n+1)(2n+1)}{6} = \frac{(n+1)(2n+1)}{6}.

\text{Var}[x] = \mathcal{E}[x^2] - \left( \mathcal{E}[x] \right)^2 = \frac{n^2 - 1}{12}.

4.3 \( \mathcal{E}(x+y+z) = \mathcal{E}(x) + \mathcal{E}(y) + \mathcal{E}(z)

= \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{3}{2}. \) If marginals are uniform

but, since \( x+y+z < 1.5 \) this is impossible.

5.11 \( \mathcal{E}[x^2] \) implies all 4 observations are \( \geq 0 \) 0

as 1 \( \leq 4 \) and 3 \( \leq 20 \). 0

\( 0 \) is prob \( (\exp(-\lambda))^4 \)

\( \mathcal{N} \) \( 4 \times (\exp(-\lambda))^3 \times \lambda \exp(-\lambda) \)

total prob \( (4\lambda + 1)(\exp(-\lambda))^4 \)

\( = (4\lambda + 1) e^{-4\lambda} \)
Q 11 \[ P( \text{at least 1 is successful}) = \frac{1}{3} + \frac{1}{3} - \frac{1}{3} \times \frac{1}{15} = \frac{7}{15}. \]

\# days to successful launch = \( PS \left( \frac{1}{3} \right) \)

\[ E \left[ \text{days} \right] = \frac{15}{7}. \]

Q 12 \[ x > n \iff \text{we achieve all H or all T} \]

\[ P(x > n) = \left( \frac{1}{2} \right)^n + \left( \frac{1}{2} \right)^n = 2 \left( \frac{1}{2} \right)^n \]

For \( n = 2, 3, \ldots \)

\[ P(x = n) = P(x > n - 1) - P(x > n) \]

\[ = \left( \frac{1}{2} \right)^{n-2} - \left( \frac{1}{2} \right)^{n-1} = \left( \frac{1}{2} \right)^{n-1} \]

Q 13. \[ \lambda = 120 \times \frac{1}{36} = \frac{10}{3} \]

\[ P(x = 3) = e^x \frac{(\frac{10}{3})^x}{x!} = 0.220. \]

Q 15. Sample size in a small unit pop. study:

\[ \lambda = \text{dist. in Binomial (n = 200, p = \frac{15,000}{500,000})} \]

\[ \lambda = n p = 6 \]

\[ P(x < 4) = P(x = 0) + P(x = 1) + P(x = 2) + P(x = 3). \]

\[ = 0.0025 + 0.0149 + 0.0446 + 0.0892 = 0.1512. \]

Q 20. \[ x \sim \text{Binom (n, p)}. \]

\[ y \sim \text{Negative Binom (r, p)} \quad \text{r pos. integer} \]

\[ P(x < r) = P(x = n - r) \]

\[ \text{more than n-r failures} \]

\[ \text{less than r successes} \]

\[ \text{successes in n Bernoulli trials} \]

\[ \text{more than n trials needed to get r successes}. \]