Class 15

AMS-UCSC

Th Mar 07, 2012
Topics
We will talk about...

1 Inference and assessing convergence
Difficulties of inference from iterative simulations.

- If the iterations have not proceeded long enough, the simulations may be underrepresentative of the target distribution relative to an independent sample of the same size.
- Within-sequence correlation. The "effective" number of simulation draws could be much smaller than the actual number of simulations.
Difficulties of Inference

Iterative simulations

Possible ways to handle the difficulties

- Simulating multiple sequences with starting points dispersed throughout the parameter space.
- Monitor convergence of all quantities of interest by comparing the variation within and between simulation sequences.
- If the simulation efficiency is very low, the algorithm can be altered (reparameterization, building efficient jumping rules).
Difficulties of Inference

Iterative simulations

- Discard early iterations to diminish the effect of the starting distribution.
- Use a subsample of the simulations, i.e. sample every $k$th simulation draw for some $k$ in order to obtain approximately independent draws from the target distribution.
- Run multiple chains, say $m$ chains, with overdispersed starting points. Monitor the convergence of the chain.
  - Monitoring scalar estimands. Monitor each scalar estimand or other quantities of interest separately. Estimands: parameters of interest, functions of two or more parameters, the value of a predicted future observation, etc. It is a good idea to transform the estimands to be approximately normal (for example, take the log of all the positive quantities and the logits of quantities in $[0, 1]$).
  - Monitoring convergence for the entire distribution.
Difficulties of Inference

Iterative simulations

Monitoring convergence of each scalar estimand.
For each scalar estimand $\psi$, we have draws from $J$ parallel sequences of length $n$ as $\psi_{ij}$. Compute the between and within-sequence variances,

$$
B = \frac{n}{J-1} \sum_{j=1}^{J} (\bar{\psi}.j - \bar{\psi}..)^2, \quad W = \frac{1}{J} \sum_{j=1}^{J} s_j^2,
$$

with $s_j^2 = \sum_{i=1}^{n} (\psi_{ij} - \bar{\psi}.j)^2 / (n-1)$. We can estimate $\text{var}(\psi|y)$ by a weighted average of $W$ and $B$,

$$
\hat{\text{var}}^+(\psi|y) = \frac{n-1}{n} W + \frac{1}{n} B.
$$

This overestimates the marginal posterior variance assuming the starting distribution is appropriately overdispersed, but it is unbiased under stationarity or when $n \rightarrow \infty$. 
Difficulties of Inference

Iterative simulations

For any finite $n$, $W$ should be an *underestimate* of $\text{var}(\psi|y)$, but in the limit, as $n \to \infty$, the expectation of $W$ approaches $\text{var}(\psi|y)$. Convergence is monitored by estimating the factor by which the scale of the current distribution for $\psi$ might be reduced if the simulations were continued in the limit $n \to \infty$. The potential scale reduction is estimated by

$$\sqrt{\hat{R}} = \sqrt{\frac{\hat{\text{var}}^+(\psi|y)}{W}},$$

which goes to 1 as $n \to \infty$. If the potential scale reduction is high, further simulations may improve the inference.
Difficulties of Inference
Iterative simulations

Monitoring convergence for the entire distribution.
Compute the potential scale reduction for all scalar estimands of interest, if \( \sqrt{\hat{R}} \) is not near to 1 for all of them, continue the simulation runs (maybe after altering the algorithm to make it more efficient). Once this quantity is near to 1 for all scalar estimands, collect the \( J \times n \) samples from the second halves of the sequences together and treat them as samples from the target distribution.
Thanks for your attention ...