1. Consider the (linear) differential equation

\[ y' + y = 3 \sin 2t. \]

(a) Use MATLAB to draw a direction field for this equation in the range \(-1 \leq t \leq 5\) and \(-3 \leq y \leq 3\) (with steps of size 0.2 for both variables).

(b) Find the explicit solutions of this differential equation satisfying the initial conditions

i. \( y(0) = -1 \)

ii. \( y(0) = 0 \)

iii. \( y(0) = 1 \)

and use MATLAB to draw their graphs in the same plot as the direction field from (a).

2. Consider the linear differential equation

\[ y' + y = \frac{7 \sin t}{\sqrt{t+1}}. \]

(a) Use MATLAB to draw a direction field for this differential equation in the range \(0 \leq t \leq 10\) and \(-3 \leq y \leq 5\) (with steps of size 0.2 for both variables).

(b) Find the solution to this differential equation that satisfies the initial condition \( y(0) = 1 \). Your answer should be expressed in terms of a definite integral of the form

\[ \int_0^t \phi(u) du \]

(and other explicit functions).

(c) Use the MATLAB function 'integral' to find \( y(5) \) and \( y(10) \).

(*) (Extra credit) Compute \( y(t) \) for \( t = 0, 0.1, 0.2, 0.3, \ldots, 9.9, 10 \), and use the result to plot the graph of \( y(t) \) in the same figure as the direction field from (a).

Suggestion: Use a 'for-loop' to compute the 101 values you need. I.e., something like

\[
\begin{align*}
&>> \text{for } j=1:101 \\
&\quad y(j)= (\text{MATLAB expression involving 'integral'}); \\
&\text{end;}
\end{align*}
\]