1. (6 pts) The population $B$ of bass in a lake is described by the model $\frac{dB}{dt} = 0.5B \cdot \left(1 - \frac{B}{20}\right)$, with $B$ measured in 1000s of bass and $t$ measured in years.

Identify the equilibrium solution(s) of this differential equation and classify them as asymptotically stable or unstable. Justify your answer(s) briefly.

- **Equilibrium solutions:** $F(B) = 0.5B \cdot \left(1 - \frac{B}{20}\right) = 0 \iff B = 0$ or $B = 20$, so the equilibrium solutions are $B_0 = 0$ and $B_1 = 20$.

- **Stability:** The quadratic function $F(B) = 0.5B \cdot \left(1 - \frac{B}{20}\right)$ is positive when $0 < B < 20$ and is negative when $B < 0$ or $B > 20$. 

![Graph showing the function $F(B) = 0.5B(1-B/20)$ with regions labeled $F<0$ and $F>0$.]
Hence, if $B(0) < 0$, then $dB/dt < 0$ and the solution decreases away from the equilibrium solution $B_0 = 0$. If $0 < B(0)$, but $B(0) < 20$, then $dB/dt > 0$ and $B(t)$ increases away from $B_0 = 0$. This means that $B_0 = 0$ is an *unstable* equilibrium.

If $0 < B(0) < 20$, then $dB/dt > 0$ and $B(t)$ increases towards $B_1 = 20$ and if $B(0) > 20$, then $dB/dt < 0$ and $B(t)$ decreases towards $B_1 = 20$. This means that $B_1 = 20$ is an *asymptotically stable* equilibrium.
2. (4 pts) When the bass population reaches 8000, anglers begin harvesting bass from the lake at a rate of 1600 fish per year. What happens to the bass population in the long run? Justify your answer by modifying the model in 1. and analyzing the new equilibrium solutions. (Pay attention to the units!)

- **New model:** Since the bass population is being measured in 1000s, the harvesting constant is $h_0 = 1.6$, and the bass population in the lake is now modeled by the differential equation
  \[
  \frac{dB}{dt} = H(B) = 0.5B \left(1 - \frac{B}{20}\right) - 1.6 = -\frac{1}{40}B^2 + \frac{1}{2}B - 1.6.
  \]

- **New equilibria:** The equilibrium solutions of the harvesting model correspond to the zeros of the function $Q(B) = -\frac{1}{40}B^2 + \frac{1}{2}B - 1.6$, which we find using the quadratic formula:
  \[
  Q(B) = 0 \implies B = \frac{-0.5 \pm \sqrt{0.25 - 0.16}}{-1/20} = 10 \pm 20\sqrt{0.09} = 10 \pm 6.
  \]
  So the equilibrium solutions are $B_1 = 4$ and $B_2 = 16$

- **Stability:** The graph of $Q(B)$ is a parabola that opens downward (because the coefficient of $B^2$ is negative), and therefore $Q(B) > 0$ if $4 < B < 16$ and $Q(B) < 0$ if $B < 4$ or $B > 16$. The same analysis we used in the first part shows that $B_1 = 4$ is an unstable equilibrium and $B_2 = 16$ is asymptotically stable.

- **Conclusion:** Since $B(0) = 8 > 4$, we conclude that in the long run, with the constant harvesting of 1600 fish per year, the bass population in the lake will stabilize at about 16,000 fish.