AMS 211 FINAL EXAM

INSTRUCTIONS:
– Answer ALL the questions, or as many as you can.
– Write your answers in the space provided on these sheets.
– SHOW ALL YOUR WORK on these sheets.

Name: ________________________________

For instructor use only:

SCORES:
Question 1: _________ (out of 20)
Question 2: _________ (out of 20)
Question 3: _________ (out of 20)
Question 4: _________ (out of 20)
Question 5: _________ (out of 20)

Total: _________ (out of 100)
1. [20 points] Determine whether the following equation is an exact differential or not and solve it accordingly

\[
\frac{dy}{dx} = -(x + y) \sin y
\]
\[
\frac{dx}{x \sin y + \cos y}.
\]

**Solution:**
2. [20 points] Solve the following homogeneous, second-order ordinary differential equation,

\[
\frac{d^2y}{dx^2} - 2 \frac{dy}{dx} + 5y = 0,
\]

subject to boundary conditions

\[
y(0) = 1, \quad \frac{dy}{dx}(0) = 0
\]

**Solution:**
3. [20 points] Find the eigenvalues and eigenvectors of

\[ A = \begin{pmatrix} 3 & -2 \\ 4 & -1 \end{pmatrix} \]

Solution:
4. [20 points] Using vector methods with a parameterization in spherical polar co-ordinates \((r, \theta, \phi)\) (where \(r\) is radius, \(\theta\) is the co-latitude measured from the North pole, and \(\phi\) is the longitude), prove that a surface element \(dS\) on a spherical surface of radius \(r = 2\) is given by

\[
dS = 4\sin \theta \, d\theta \, d\phi
\]

Hence show that

\[
\int_S z \, dS = 8\pi
\]

where \(S\) is the upper (Northern) hemisphere.

**Solution:**

...
5. [20 points] Find an analytic function $f(z) = u(x,y) + iv(x,y)$ of the complex variable $z = x + iy$ that has real part

$$u(x,y) = x^3 - 3xy^2$$

Note: the answer must be written as a function of $z$ not as a function of $x$ and $y$

**Solution:**