percentiles.

\( Q_1 \) - 1st quartile.
\( Q_3 \) - 3rd quartile.

\[ IQR = Q_3 - Q_1 \]

Inter quartile range.
- measure of spread that is more robust to the presence of outliers
Quantifying chance.

- Mathematics to calculate quantities associated with uncertain events.
- What do we mean by chance?

If a balanced die is rolled many times, each side should come up an often as any of the others.
- The side with 2 dots should come up $\frac{1}{6}$ of the time.

**Definition:** The chance of an event is the percentage of times the event is expected to happen when the process is repeated over and over independently and under the same conditions.
Frequency definition doesn't work for events that cannot be repeated

eg what's the chance of finding life on Mars in the next 10 years?

The subjective interpretation of probability can be used in this type of situation.

independently - outcome of one trial doesn't depend on earlier trials.

same conditions - aspect of repeatability.

Properties

If something is impossible, it happens 0% of the time, so its probability is zero.

If something always happens, it happens 100% of the time, its probability is one.
chances are between 0 and 1.

0% 100%

chances of something happening or it not happening must be 100%

by roll a 3 or roll a not 3 (ie 1, 2, 4, 5, 6).

one of these must happen.

chance of an event is 100% = the chance of the opposite event

or:

if an event has a chance $p$ of happening

the opposite event has chance $1 - p$
Example.

Box contains red and blue marbles.

Draw one "at random" - each marble is equally likely to be chosen.

If it's red - win $1.
   blue - win nothing

Box 1: 3 red  2 blue
Box 2: 30 red  20 blue.

Your chance of winning is the same for each box.

- From the definition repeat the process independently under the same conditions.
  ie draw a marble repeatedly.

In both cases, get a red marble 3 out of 5 the ratio is what's important.
In general, for equally likely outcomes, the probability of an event is

\[
\frac{\text{# outcomes that are part of the event}}{\text{total # outcomes}}.
\]

List all the possible outcomes.

Box A.

R  
R  
R  
G  
G  

Count the # that fit the condition

3

Divide by total # outcomes to get probability.

p = \frac{3}{5}
What's the probability that there will be a quiz today?

- printed syllabus says yes.
- a line says nothing.

50% - each outcome is equally likely.

My probability is that the probability of a quiz today is $\theta$.

The course website should be considered more accurate than the printed syllabus.

But this is a neat example of using probability to encode knowledge (my knowledge was different to yours) and where the frequency definition is un-natural ("repeat today's class over and over and count the # of times there was a quiz.")
Drawing with and without replacement.

With replacement:

- Draw one, note the colour, put it back.
- When we draw the 2nd ball, the contents of the bag are the same.

Without replacement:

- Imagine drawing B.
- When drawing 2nd ball, contents have changed.
- Therefore, chances of drawing each ball have changed.

Need conditional probability for this type of situation.
Conditional Probabilities

The occurrence of one event affects the chance of another event.

E.g. if 1st ball is blue

Chance of second ball being green is different than if the 1st ball was green.

Cards are shuffled

Deal 2 cards face down.

Q: What is the chance that the 2nd card is a 4?

Shuffling 52 cards at random cards.

So 2nd card is equally likely to be any one of the 52 cards.

Equivalently

The 4th is equally likely to be in any one of the 52 positions.
Conditional Probability

The occurrence of one event affects the chance of another event.

E.g. If 1st ball is blue, chance of 2nd ball being green is different than if the 1st ball was green.

E.g. Standard 52 card deck

Shuffle cards
Deal 2 cards face down.

→ What is the chance that the 2nd card is Q♥?

Shuffling puts the cards in random order. 2nd card is equally likely to be any one of the 52 cards equivalently:
Q♥ is equally likely to be in any one of the 52 positions.
chance of the 2nd card being \( Q \heartsuit \)

\[
= \frac{1}{52}.
\]

- equally likely outcomes

\[
\frac{\text{# outcomes we're interested in}}{\text{total # outcomes}}.
\]

Q: Turn over 1st card. (and find it is not \( Q \heartsuit \)).
what's the chance that 2nd card is \( Q \heartsuit \)

Now, there are only 51 places the \( Q \heartsuit \)
could be.

\[
\Rightarrow \text{probability that 2nd card is } Q \heartsuit \text{ is now } \frac{1}{51} \quad \left( \frac{\text{# outcomes of interest}}{\text{total # outcomes}} \right)
\]

this is the conditional probability of the 2nd card being \( Q \heartsuit \) given 1st card being \( A \spadesuit \).

(conditioned on)
Multiplication Rule.

Draw 2 balls without replacement.

What are the chances of drawing
1st (R)
2nd (G)

1st approach.

Imagine a large number of people doing this (say 600).

About \( \frac{1}{3} \) (\( \approx 200 \) people) will get (R) on 1st draw.

Of these people, about \( \frac{1}{2} \) will get (G) on 2nd draw.

\( \Rightarrow \) 100 people will get (R) then (G).

\( \Rightarrow \) Chances of drawing (R) then (G) is \( \frac{100}{600} = \frac{1}{6} \).
2nd approach.

Initially \[ \begin{array}{ccc} R & G & B \end{array} \]

1st draw, each ball has equal chance to be picked
\[ \Rightarrow \text{prob. of } R = \frac{1}{3} \]

Conditioned on drawing \( R \), box now contains
\[ \begin{array}{cc} G & B \end{array} \]

prob of \( G \) is \( \frac{1}{2} \).

\( \frac{1}{2} \) of the time that we succeed on the 1st trial do we succeed on the 2nd trial.
\[ \Rightarrow \frac{1}{2} \times \frac{1}{3} = \frac{1}{6} \] of the time we get \( G \), \( G \).
**Approach 3:**

<table>
<thead>
<tr>
<th>1st draw</th>
<th>2nd draw</th>
<th>Final outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>G</td>
<td>RG</td>
</tr>
<tr>
<td>R</td>
<td>B</td>
<td>RB</td>
</tr>
<tr>
<td>G</td>
<td>B</td>
<td>GB</td>
</tr>
<tr>
<td>B</td>
<td>G</td>
<td>BG</td>
</tr>
</tbody>
</table>

There are 6 equally likely outcomes. We are interested in 1 of them.

\[
\text{prob. red then green} = \frac{1}{6}.
\]

The probability that two events will happen equals the probability that the first will happen times the probability that the second will happen given that the first has happened.
product of prob. of 1st event and conditional prob. of 2nd event given 1st.