4 all the ways of getting at least one 3.

What do we know about these possibilities?

- mutually exclusive.

prob. that one of the events occur?

= sum of the probability of each event.

\[ P(3---) = \frac{1}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6}, \]
\[ P(-3--) = \frac{5}{6} \times \frac{1}{6} \times \frac{5}{6} \times \frac{5}{6}. \]
\[ P(--3-) = \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} \times \frac{5}{6}. \]
\[ P(--3) = \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \]
\[ P(33--) = \frac{1}{6} \times \frac{1}{6} \times \frac{5}{6} \times \frac{5}{6}. \]
Can we avoid counting
Can we avoid explicitly listing all the components of the event we are interested in?

1 Red
9 Green

5 draws with replacement

What's the probability of exactly two of the draws being red?

One way of getting 2 red is

\[ \begin{array}{cccc}
\text{G} & \text{G} & \text{G} & \text{G} & \text{G} \\
\text{R} & \text{G} & \text{G} & \text{G} & \text{G} \\
\end{array} \]

The draws are with replacement, so the events are independent; the probability of the sequence of draws is the product of the probability of each draw.

\[
\frac{9}{10} \times \frac{1}{10} \times \frac{9}{10} \times \frac{1}{10} \times \frac{9}{10}.
\]
There are other sequences that result in 2R + 8G.

\[
\begin{array}{cccccc}
R & R & G & G & G \\
R & G & R & G & G \\
R & G & G & R & G \\
R & G & G & G & R \\
G & R & R & G & G \\
G & R & R & G & R \\
G & G & R & R & G \\
G & G & R & G & R \\
G & G & G & G & R \\
\end{array}
\]

Probability of each of them is the same.

\[
\frac{1}{10} \times \frac{1}{10} \times \frac{9}{10} = \frac{9}{10} \times \frac{9}{10}.
\]

The different sequences are mutually exclusive.

\[
\Rightarrow \text{probability of getting any one of them} = \text{sum of prob of each} = \text{# of sequences} \times \text{prob of sequence}.
\]

\[
= 10 \times \left( \frac{1}{10} \right)^2 \times \left( \frac{9}{10} \right)^3
\]
To avoid listing all the possibilities
- use Binomial coefficient.

5 trials.
2 red \( \Rightarrow \) sum to 5.
3 green.

\[
\text{binomial coefficient} = \frac{5 \times 4 \times 3 \times 2 \times 1}{(2 \times 1) \times (3 \times 2 \times 1)}
\]

= 10.

= \# of ways of getting
2 red + 3 green in 5 draws.

5 trials
1 red
4 green.

\[
\text{binomial coefficient} = \frac{5 \times 4 \times 3 \times 2 \times 1}{(1) \times (4 \times 3 \times 2 \times 1)}
\]

= 5

notation: \( n! \) " \( n \) factorial"

\( n \times (n-1) \times (n-2) \times \ldots \times 2 \times 1 \)

1! = 1.

0! = 1. \( \Rightarrow \) By definition.
$1! = 1$
$2! = 2$
$3! = 3 \times 2 \times 1 = 6$
$4! = 4 \times 3 \times 2 \times 1 = 24$
$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$

What does $n!$ give us?

It gives us the number of different ways there are to arrange $n$ objects.

In terms of counting the number of permutations, when the objects are of different colours, and the objects of a particular colour are indistinguishable, we need to factor out the permutations $e$ within each colour.
\[ \text{# ways of arranging } n \text{ objects where} \]
\[ k \text{ are of one type and } (n-k) \text{ are of a different type is} \]
\[ \frac{n!}{k! (n-k)!} \]

5 draws, 2 R, 3 G. \quad n = 5 \quad k = 2.
\[ \frac{5!}{2! \times 3!} = 10 \]

5 draws or 5 G. \quad n = 5 \quad k = 0.
\[ \frac{5!}{0! \times 5!} = 1 \quad \text{-- only one way to get 5 G.} \]

\[ \binom{n}{k} = \text{"n choose k"} \]

n is number of trials.

k is the number of "successes"
If the trials are independent and prob. of "success" is \( p \)
"failure" is \( 1-p \)

Then the prob. of \( k \) successes in \( n \) trials
is \( \binom{n}{k} p^k (1-p)^{n-k} \)

\[
\frac{n-k}{k} \frac{p^k (1-p)^{n-k}}{k} \quad \text{\( (n-k) \) failures, each with prob. \( (1-p) \).}
\]

\( k \) successes, each with prob. \( p \).

Provided that:

- \( n \) is fixed in advance.
- \( p \) is the same for all trials
- Trials are independent.
Example.

A family has 4 children.

What is the probability that they have more girls than boys?

Assume that each child is boy/girl with \( p = \frac{1}{2} \) independently.

3 out of 4 girls
4 out of 4 girls.

- These are mutually exclusive.
- \( \Rightarrow \) add probabilities.

\[
\frac{4!}{3! (4-3)!} \cdot \left( \frac{1}{2} \right)^2 \cdot \left( \frac{1}{2} \right)^{4-2} \\
+ \frac{4!}{4! (4-4)!} \cdot \left( \frac{1}{2} \right)^4 \cdot \left( \frac{1}{2} \right)^{4-4}
\]

\[
= \frac{4 \times 1}{16} + \frac{1 \times 1}{16} = 0.31.
\]

In class we have 11 families with 4 children.

4 have 3 or 4 girls.

\[
\frac{4}{11} = \text{"close enough"}
\]
Lottery:

49 numbered balls.

6 are drawn.

Jackpot if match all 6.

Probability of winning

\[
\frac{6}{49} \times \frac{5}{48} \times \frac{4}{47} \times \frac{3}{46} \times \frac{2}{45} \times \frac{1}{44} =
\]

\[
\frac{1}{13,983,816}
\]

- you are not going to win
- your choice of numbers does not affect your chance of winning.
- your choice of numbers does affect your chance of sharing the prize if you win.
Ball #23.

The chance that ball #23 is picked in a given week is $\frac{6}{49}$.

How much fluctuation in the # of times each ball is picked do we expect?

What do we mean when we say that we expect the numbers to "average out"?