$H_0$: earthquakes are uniformly distributed in time
$H_1$: no they are not.

$$\chi^2 = \text{sum} \left( \frac{(\text{observed value} - \text{expected value})^2}{\text{expected value}} \right)$$

$$= 94$$

$$\# \text{dof} = \# \text{terms} - 1 \chi^2 \text{ sum} - 1$$

$$= 7 - 1$$

$$= 6$$

$p$-value $\leq 1\% \ (4.5 \times 10^{-18})$

Thus reject $H_0$ and conclude that earthquakes prefer to happen on Thursdays & Sundays.
large earthquakes

\[ \text{(1 per year)} \]

10 years of observations with large variability
in a sample this small

after shocks come close afterwards.

\[ \text{not unreasonable to have non-uniformity in time in the observed data.} \]

Conclusion

the null hypothesis should accurately model
how we think the data is generated.

Rejecting $H_0$ should be an alert pointer that we may
need to think more carefully about what is
going on (the process behind the data)
Testing Independence

$\chi^2$ can also be used to test independence.

E.g. is handedness and sex independent?

is year and class attendance independent?

<table>
<thead>
<tr>
<th></th>
<th>Men</th>
<th>Women</th>
</tr>
</thead>
<tbody>
<tr>
<td>Right handed</td>
<td>25</td>
<td>57</td>
</tr>
<tr>
<td>Left handed</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>Ambidextrous</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

- 92

Thus is a sample from which population?

- reasonable to say that it should be representative of all people of that 18-25 y.o.

$H_0:$ sex and handedness are independent

- men/women are right handed at the same rate

Turn this into a box model that will generate the expected values
what's in the box?

6 types of ticket

- RH Men
- RH Women
- LH Men
- LH Women
- A Men
- A Women

each ticket has a 1 on it as we're counting how many of each category are in our sample.

How many of each type of ticket should we have?

\[
\text{what } \% = \frac{\text{from the }}{\text{independence}}.
\]

\[
\% \text{ RH amongst men equals } \% \text{ RH amongst women.}
\]

\[
\% \text{ of men and women on each row of the table should be the same.}
\]

\[
\% \text{ of all people who are RH} = \frac{\text{total } \# \text{ of RH people}}{\text{total } \# \text{ of people}} \times 100 = \frac{82}{92} \times 100 = 89.1 \%
\]
Expected values:

- Expect 89.1% of men to be RH.
- 89.1% of women to be RH.

\[
\frac{89.1}{100} \times 29 = 25.58
\]

\[\text{total} \quad \# \text{men}\]

\[
\frac{89.1}{100} \times 63 = 56.1
\]

In general:

\[
\text{expected value in a cell} = \frac{\text{row total} \times \text{column total}}{\text{table total}}
\]

\[
\begin{array}{c|cc}
\text{Expected values} & \text{Men} & \text{Women} \\
\hline
\text{RH} & 25.58 & 56.1 \\
\text{LH} & 2.5 & 5.5 \\
\text{A} & 0.63 & 1.37 \\
\end{array}
\]
\[ \chi^2 \text{ statistic - measures how far our data is from what we would expect, in terms of the amount of variability we would expect.} \]

\[ \chi^2 = \text{sum} \left( \frac{(\text{observed} - \text{expected})^2}{\text{expected}} \right) \]

\[ = \frac{(2.5 - 2.5)^2}{2.5} + \frac{(5.7 - 56.1)^2}{56.1} \]
\[ + \frac{(3 - 2.5)^2}{2.5} + \frac{(5 - 5.5)^2}{5.5} \]
\[ + \frac{(1 - 0.63)^2}{0.63} + \frac{(1 - 1.37)^2}{1.37} \]

\[ = 0.5 \]

\[ \text{p-value: if a have a population whose handedness and sex are independent, how likely is it that a sample of size 92 will result in a } \chi^2 \text{ value } \geq 0.5? \]
# dof

The model is not fully specified.
- We have estimated the % of each category in the population from the sample data.

Table with $m$ rows and $n$ columns there are $(m-1) \times (n-1)$ d.o.f.

In this case, $m = 3$, $n = 2$

$\# \text{dof} = \frac{(3-1)(2-1)}{(3\times2)} = 2.$

From the table p-value is between 70% and 90%.

ie in more than 70% of samples from a population with independence between sex and handedness, a sample of size 92 will result in a $X^2$ value $\geq 0.5$.

$\Rightarrow$ Cannot reject H0.
A CHI-SQUARE TABLE

The chi-square curve, with degrees of freedom shown along the left of the table.

The shaded area is shown along the top of the table.

is shown in the body of the table.

<table>
<thead>
<tr>
<th>Degrees of freedom</th>
<th>99%</th>
<th>95%</th>
<th>90%</th>
<th>70%</th>
<th>50%</th>
<th>30%</th>
<th>10%</th>
<th>5%</th>
<th>1%</th>
</tr>
</thead>
<tbody>
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<td>0.0039</td>
<td>0.016</td>
<td>0.15</td>
<td>0.46</td>
<td>1.07</td>
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<td>0.10</td>
<td>0.21</td>
<td>0.71</td>
<td>1.39</td>
<td>2.41</td>
<td>4.60</td>
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<td>9.21</td>
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<tr>
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<td>0.35</td>
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<tr>
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<td>0.71</td>
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<td>2.20</td>
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<td>9.49</td>
<td>13.28</td>
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<td>1.61</td>
<td>3.00</td>
<td>4.35</td>
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<td>22.78</td>
<td>28.41</td>
<td>31.41</td>
<td>37.57</td>
</tr>
</tbody>
</table>

A few comments about tests of significance.

1. The 5% and 1% thresholds
   ("statistically significant" and "highly statistically significant") are arbitrary but conventional.
   - don't stress over the difference between 4.9 and 5.1%

2. Recall what a p-value is
   - chance of observing data as extreme or more than the data you have.

3. If you do enough tests, eventually you will get a significant result.

Decide how you are going to analyse the data before you collect it.
- avoid multiple hypothesis tests based on seeing the data.
Cancer clusters at phone masts

Daniel Foggo

SEVEN clusters of cancer and other serious illnesses have been discovered around mobile phone masts, raising concerns over the technology's potential impact on health.

Studies of the sites show high incidences of cancer, brain haemorrhages and high blood pressure within a radius of 400 yards of mobile phone masts.

One of the studies, in Warwickshire, showed a cluster of 31 cancers around a single street. A quarter of the 30 staff at a special school within sight of the 90ft high mast have developed tumours since 2000, while another quarter have suffered significant health problems.

The mast is being pulled down by the mobile phone after the presentation of the evidence. The O2 group local protesters. While rejecting any links to ill-health, O2 admitted the decision was "clearly rare and unusual".

Phone masts have provoked protests throughout Britain with thousands of people objecting each week to planning applications. There are about 47,000 masts in the UK.

Dr John Walker, a scientist who compiled the cluster studies with the help of local campaigners in Devon, Lincolnshire, Staffordshire and the West Midlands, said he was convinced they showed a potential link between the angle of the beam of radiation emitted from the mast's antennae and illnesses discovered in local populations.

"Masts should be moved away from conurbations and schools and the power turned down," he said.

Some scientists already believe such a link exists and studies in other European countries suggest a rise in cancers close to masts. In 2005 Sir William Stewart, chairman of the Health Protection Agency, said he found four such studies to be of concern but that the health risk remained unproven.

HAVE YOUR SAY

there are enough open spaces around the world to put these masts up so why put them in populated areas if there is the slightest doubt about them causing ill health
Jane motter, Chester,

Is there any evidence to say that these people wouldn't have got ill if the mobile phone mast had never been put up??
Jasmine, Boston,

There are 3 masts on top of our local hospital which is next to my school! Shows how much they think about it when put them up!!
Beth, Boston,

Read all 85 comments

HAVE YOUR SAY
PRINT EMAIL POST TO BLOGS
POST TO FARK POST TO YAHOO! POST TO DIGG
p-value: chance of observing data, or more extreme data, assuming H₀ is true.

How many phones must we have?
- many 10's of thousands.

If chose 10,000 random locations, how many of them do we expect will be within 400 yards of a cluster of cancers?
One sided vs two sided tests.

depending on the alternative hypothesis
deviations in one or both directions from
the expected value under Ho can constitute
evidence against Ho.

Ho - coin is unbiased

H1 - coin is biased towards Heads

H1 - coin is biased

4/ The difference between "statistically significant"
and "important"

$Z$ statistic = $\frac{\text{observed} - \text{expected}}{\text{SE}}$

%'s. $SE = \frac{SD \text{bar} \times 100}{\sqrt{\text{# draws}}}$

as # draws increases, SE% decreases.
Assume a small difference between observed $z$ expected.

as $\#$ draws increases, $z$ increases and eventually gives a p-value $< 5\%$.

Example:

Reading scores

Rural scores mean 25
Urban 26

SD of scores were 10
N = 2,500 in both regions.

2-sample $z$-test

H0: reading scores are same in rural + urban areas.

$$ z = \frac{(25 - 26) - 0}{SE_{d}} $$

$SE_{d} = \frac{10}{\sqrt{\frac{50}{2}}} = 0.2$

$SE_{d} = \sqrt{\left(SE_{1}\right)^{2} + \left(SE_{2}\right)^{2}} = \sqrt{0.2^{2} + 0.2^{2}} = 0.28$

$$ z = \frac{-1}{0.28} = -3.5 $$

p-value = 100 - 99.95 (from table; 2-sided test means or "different")
p-value << 1%.

Reject Ho that reading scores are the same.

But what does a difference of 1 point in reading scores mean?

- Partial understanding of 1 word in list
- A list of 40 words.

Is this important?

Could something else be going on?

- Earthquake distribution in time
- ESP

- What is stated as Ho is not actually Ho.

Sometimes you don't need a randomized controlled trial
- Parachutes
- Early knee surgery.