Chap 13

Exercise set C

1. (a) \( \Pr(\text{first roll} = \square) = \frac{1}{6} \)
   (b) \( \Pr(1^{st} = \square, 2^{nd} = \spadesuit, 3^{rd} = \clubsuit) = \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{216} \) (each roll is independent)

2. It is legitimate

Exercise set D

3. 1 yr = 52 weeks \(\Rightarrow\) 10 yrs = 520 weeks
   this is independent events

   not winning each week = \( \frac{999,999}{1,000,000} \)

   not winning 10 yrs = \( \left( \frac{999,999}{1,000,000} \right)^{520} \approx 0.9995 \)

4. The 2 draws are dependent, because this is drawing without replacement

Review exercise

1 2 3 4

5. (i) club diamond heart spade

(ii) 1 2 3 4 different suits

Option (ii) is better, it does not consider the "ordering"
\[ \frac{4}{52} \times \frac{3}{51} \times \frac{2}{50} \times \frac{1}{49} \times \frac{4}{48} = \frac{1}{3,246,720} \]

\[ 1 \ 1 \ 8 \ \boxed{11/8} \]

are color and number independent?

Yes!

\[ P(\text{8 and black}) = \frac{1}{6} \]

\[ = P(\text{8}) \times P(\text{black}) = \frac{2}{6} \times \frac{1}{2} = \frac{1}{6} \]

---

Chap 14

Exercise set C

(4) \[ \text{unconditional} \quad P(A) = \frac{1}{2} \]

\[ \text{unconditional} \quad P(B) = \frac{1}{3} \]

(a) \[ P(A \cap B) = \frac{1}{6} \quad \text{false, need to consider whether A and B are dependent or not} \]

(b) if A and B independent, \[ P(A \cap B) = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6} \], true

(c) if A and B mutually exclusive, \[ P(A \cap B) = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6} \], false, \[ P(A \cap B) = 0 \]

(d) \[ P(A \cup B) = \frac{1}{2} + \frac{1}{3} = \frac{5}{6} \], false, should consider dependency

(e) if A and B independent, \[ P(A \cup B) = \frac{1}{2} + \frac{1}{3} = \frac{5}{6} \], false,

\[ P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{1}{2} + \frac{1}{3} - \frac{1}{6} = \frac{5}{6} - \frac{1}{6} = \frac{2}{3} \]

(f) if A and B mutually exclusive, \[ P(A \cup B) = \frac{1}{2} + \frac{1}{3} = \frac{5}{6} \], true
2 cards dealt off the top of well-shuffled deck

(a) \( \Pr(2^\text{nd} = \text{ace}) = \frac{4}{52} = \left(\frac{4}{52}\right) \times \left(\frac{4}{52}\right) \times \left(\frac{4}{51}\right) = \frac{1}{13} \)

(b) \( \Pr(2^\text{nd} = \text{ace} \mid 1^\text{st} = \text{king}) = \frac{4}{52} \times \frac{4}{51} = \frac{4}{663} \)

(c) \( \Pr(1^\text{st} = \text{king}, 2^\text{nd} = \text{ace}) = \frac{4}{52} \times \frac{4}{51} = \frac{4}{663} \)

**Review exercise**

(a) if A and B are independent, then they are mutually exclusive. **False**  \( P(A \cap B) = P(A) \cdot P(B) = \frac{1}{5} \times \frac{1}{10} = \frac{1}{50} \), not mutually exclusive.

(b) if A and B are mutually exclusive, then they are not independent **true**

4 draws

\[ P(\text{z is drawn at least once}) = 1 - P(\text{z is never drawn}) \]

\[ = 1 - \left(\frac{3}{5}\right)^4 = \frac{544}{625} = 0.8704 \]

chap 15

die rolled 10 times, \( P(\text{it never lands six}) = \left(\frac{5}{6}\right)^{10} \)
family of 4 child, what proportion have more girls than boys

\[\begin{array}{c|cc}
\text{gender} & \text{count} & \text{probability} \\
\hline
0g & 4b & \left(\begin{array}{c} 4 \\ 0 \end{array}\right) \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^0 \\
1g & 3b & \left(\begin{array}{c} 4 \\ 1 \end{array}\right) \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^3 \\
2g & 2b & \left(\begin{array}{c} 4 \\ 2 \end{array}\right) \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^2 \\
3g & 1b & \left(\begin{array}{c} 4 \\ 3 \end{array}\right) \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^1 \\
4g & 0b & \left(\begin{array}{c} 4 \\ 4 \end{array}\right) \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^0 \\
\hline
\end{array}\]

\[P(\text{more girls than boys}) = \frac{\left(\begin{array}{c} 4 \\ 3 \end{array}\right) + \left(\begin{array}{c} 4 \\ 2 \end{array}\right) + \left(\begin{array}{c} 4 \\ 1 \end{array}\right) + \left(\begin{array}{c} 4 \\ 0 \end{array}\right)}{\left(\begin{array}{c} 4 \\ 1 \end{array}\right) + \left(\begin{array}{c} 4 \\ 2 \end{array}\right) + \left(\begin{array}{c} 4 \\ 3 \end{array}\right) + \left(\begin{array}{c} 4 \\ 4 \end{array}\right)} = \frac{4+1}{1+4+6+4+1} = 0.3125\]

\[\binom{8}{2} = 28 \quad \text{first list}\]

\[\binom{8}{5} = 56 \quad \text{second list}\]

therefore, it is true, second list larger than first list

\[\begin{array}{c|c}
\text{color} & \text{count} \\
\hline
r & 6b & 6b & 6b & 6b & 6b \\
\hline
\end{array}\]

\[P(\text{exactly 2 draws will be red}) = \binom{5}{2} \cdot \left(\frac{1}{10}\right)^2 \cdot \left(\frac{9}{10}\right)^3 = 10 \cdot \left(\frac{1}{10}\right)^2 \cdot \left(\frac{9}{10}\right)^3\]

this is true, we are not using addition rule.