Chapter 7:

Section 7-2: 8, 12, 14, 18, 23, 24, 27, 30, 35, 36, 39.

8. Let \( p \) be the population proportion of men who watch golf on TV. The claim is that this proportion is not 0.7 (70% as expressed as a decimal), or \( p \neq 0.7 \). If this is true then \( p = 0.7 \) must be false. Since \( p = 0.7 \) includes equality, we let \( H_0 \) be \( p = 0.7 \) and we let \( H_1 \) be \( p \neq 0.7 \).

12. Let \( \sigma \) be the population standard deviation of women biologists’ salaries. The claim is that this standard deviation is greater than 3,000, or \( \sigma > 3,000 \). If this is true then \( \sigma \leq 3,000 \) must be false. Since \( \sigma \leq 3,000 \) includes equality, we let \( H_0 \) be \( \sigma = 3,000 \) and we let \( H_1 \) be \( \sigma > 3,000 \).

14. In a two-tailed test, the critical values are \( \pm z_{\alpha/2} \). Since \( \alpha = 0.01 \), \( \alpha/2 = 0.005 \). The critical values are then \( \pm z_{0.005} = \pm 2.575 \).

18. Since \( H_1 \) is \( p > 0.18 \), this is a right-tailed test. In a right-tailed test, the critical value is \( z_{\alpha} \). Since \( \alpha = 0.10 \), the critical value is \( z_{0.10} = 1.28 \).

24. \( H_0 : p = 0.103 \), which makes \( q = 0.897 \), \( n = 800 \), and \( \hat{p} = 0.12 \). The test statistic is then

\[
 z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{0.12 - 0.103}{\sqrt{\frac{0.103 \cdot 0.897}{800}}} = 1.582
\]

30. Since \( H_1 : p \neq 0.30 \), it is a two-tailed test, and the test statistic \( z = 2.44 \), is to the right of center so the p-value is the twice the area to the right of the test statistic. Using the methods of 5-2, the P-value is \( 2 \times (1 - 0.9927) = 0.0146 \).

36. The original claim did include equality, \( H_0 \) was rejected, so the conclusion is: There is sufficient evidence to warrant the rejection of the claim that the proportion of M&Ms that are blue is equal to 0.10.
Section 7-3: 2, 9, 11, 12, 15.

2. **Survey of Drinking**
   a. The sample proportion is \( \hat{p} = 0.62 \), the claim is that the proportion is 0.50, so \( p = 0.50 \). The sample size is \( n = 1087 \). The requirements are met, since \( np = 1087 \times 0.50 = 543.5 \) and \( nq = 1087 \times 0.50 = 543.5 \). The test statistic is
   \[
   z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{0.62 - 0.50}{\sqrt{\frac{0.50 \times 0.50}{1087}}} = 7.913
   \]
   b. The significance level is 0.05, and the claim is that the proportion greater than 0.50, making this a right-tailed test. The critical value is \( z_{0.05} = 1.645 \).
   c. Since this is a right-tailed test with test statistic to the right of center, the P-value is the area to the right of the test statistic. This area is found using Table A-2, and is 0.0001. So, \( P-value = 0.0001 \).
   d. Since the P-value is smaller than the significance level, we would reject \( H_0 \). The conclusion is: The sample data support the claim that the majority of adults use alcoholic beverages.
   e. No. Small sample sizes in similar tests could have results that do not cause us to reject \( H_0 \).

12. **Smoking and College Education**
    First, we check the requirements. The sample is a random sample. The conditions for a binomial are satisfied. The sample size is 785 with the claim that the percentage of people with four years of college that smoke is less than 27%, making \( p = 0.27 \), so \( np = 785 \times 0.27 = 211.95 \) and \( nq = 785 \times 0.73 = 573.05 \). The requirements are satisfied.
    The claim is that the percentage of people with four years of college that smoke is less than 27%, so this is a left-tailed test. In the sample, 144 subjects with four years of college education smoked. The sample proportion is \( \hat{p} = x/n = 144/785 = 0.183 \).
    \( H_0: p = 0.27 \)
    \( H_1: p < 0.27 \)
    The test statistic is
    \[
    z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{0.183 - 0.27}{\sqrt{\frac{0.27 \times 0.73}{785}}} = -5.490
    \]
    In a left-tailed test at the 0.01 significance level, the critical value is \( -z_{0.01} = -2.33 \).
    Since this is a left-tailed test, the P-value is the area to the left of the test statistic. Using Table A-2, we find that the \( P-value = 0.0001 \).
    We reject the null hypothesis.
    The sample data support the claim that the percentage of people with four years of college that smoke is less than 27%. One reason that they may smoke at a lower rate is that they are better educated on the hazards and dangers.

Section 7-4: 3, 4, 7, 8, 9, 12.

4. Since \( \sigma \) is known, and the sample size (\( n = 47 \)) is larger than 30, the conditions are met.
8. The test statistic is \( z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{45,678 - 46,000}{9900 / \sqrt{65}} = -0.262 \)

The claim is that the mean salary for college graduates who have taken a statistics course is equal to $46,000, making this a two-tailed test. In a two-tailed test at the 0.05 significance level, the critical values are \( \pm z_{0.025} = \pm 1.96 \).

Since this is a two-tailed test and the test statistic is to the left of center, the P-value is twice the area to the left of the test statistic. Using Table A-2, we find that the \( P\text{-value} = 2 \times (0.0974) = 0.1948 \).

There is not sufficient evidence to warrant the rejection of the claim that the mean salary for college graduates who have taken a statistics course is equal to $46,000.

12. Head Circumferences

First, we check the conditions. The sample size \( (n = 100) \) is greater than 30, we are assuming a value of \( \sigma \), and the sample is a random sample. The claim is that the mean head circumference for two month old babies is 40.0 cm, so this is a two-tailed test. The sample mean is 40.6 cm and the assumed population standard deviation is 1.6 cm. The significance level is 0.05.

\[ H_0: \mu = 40.0 \]
\[ H_1: \mu \neq 40.0 \]

The test statistic is \( z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{40.6 - 40.0}{1.6 / \sqrt{100}} = 3.75 \)

In a two-tailed test at the 0.05 significance level, the critical values are \( \pm z_{0.025} = \pm 1.96 \).

Since this is a two-tailed test and the test statistic is to the right of center, the P-value is twice the area to the right of the test statistic. Using Table A-2, we find that the \( P\text{-value} = 2 \times (1 - 0.9999) = 0.0002 \).

We reject the null hypothesis.

There is sufficient evidence to warrant the rejection of the claim that the mean head circumference for two month old babies is 40.0 cm.

Section 7-5: 3, 4, 5, 6, 11, 12, 16, 17, 19.

4. The data appear to come from a distribution that is not normal, the sample size, \( n = 150 \), is larger than 30, and \( \sigma \) is unknown, so this involves the Student t distribution.

6. In the row for 11 degrees of freedom, the absolute value of the test statistic falls to the right of 1.363 so the P-value is more than 0.10.

12. The claim is that the mean starting salary for college graduates who have taken a statistics course is equal to $46,000, so this is a two-tailed test. The sample size is \( n = 27 \), the sample mean is \( \bar{x} = 45,678 \), and the sample standard deviation is \( s = 9,900 \). The significance level is 0.05.

The test statistic is \( t = \frac{\bar{x} - \mu}{s / \sqrt{n}} = \frac{45,678 - 46,000}{9,900 / \sqrt{27}} = -0.169 \)

In a two-tailed test at the 0.05 significance level with 26 degrees of freedom, the critical values are \( \pm t_{0.025} = \pm 2.056 \).

Since the absolute value of the test statistic is smaller than 1.315, the P-value is greater than 0.20.

We fail to reject the null hypothesis.

There is not sufficient evidence to warrant the rejection of the claim that the mean salary for college graduates who have taken a statistics course is equal to $46,000.
16. Monitoring Lead in Air
The claim is that the amount of lead in the air in Building 5 of the World Trade Center following the collapse of the two buildings is greater than 1.5 $\mu g/m^3$, so this is a right-tailed test. The sample size is $n = 6$ making the degrees of freedom $df = 5$. The significance level is 0.01. Calculating $\bar{x}$ from the sample data gives
$$\bar{x} = \frac{\sum x}{n} = \frac{9.23}{6} = 1.538$$
Calculating the sample standard deviation from the data gives
$$s = \sqrt{\frac{n\sum(x^2) - (\sum x)^2}{n(n-1)}} = \sqrt{\frac{6 \cdot 32.520 - (9.23)^2}{6 \cdot 5}} = 1.914$$
We move on to the test.
$H_0: \mu = 1.5$
$H_1: \mu > 1.5$

The test statistic is
$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{1.538 - 1.5}{1.914/\sqrt{5}} = 0.044$$
In a right-tailed test at the 0.01 significance level with $df = 5$, the critical value is $t_{0.01} = 3.365$.
In the row for 5 degrees of freedom, the test statistic lies below 1.476, so the $P$-value is greater than 0.10.
We fail to reject the null hypothesis.
There is not sufficient sample evidence to support the claim that the amount of lead in the air in Building 5 of the World Trade Center following the collapse of the two buildings is greater than 1.5 $\mu g/m^3$.
The assumption that the data came from a normally distributed population seems unrealistic, as there is an extreme outlier in the data set.

Section 7-6: 2, 3, 6, 7, 9.

2. The test statistic is
$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{4 \cdot 18^2}{12^2} = 9.0$$
The alternate hypothesis is $H_1: \sigma > 15$, so this is a right-tailed test. Since $n = 5$, $df = 4$, and the significance level is 0.01. The critical value, from Table A-4 is 13.277. Since the test statistic falls between 7.779 and 9.488, the $P$-value is between 0.05 and 0.10. There is not sufficient evidence to support the alternate hypothesis.

6. Supermodel Weights
The claim is that the population standard deviation of supermodel body weights is less than 29lb, so this is a left-tailed test. The sample size is $n = 9$ making the degrees of freedom $df = 8$. The significance level is 0.01. Calculating the sample standard deviation from the data gives
$$s = \sqrt{\frac{n\sum(x^2) - (\sum x)^2}{n(n-1)}} = \sqrt{\frac{9 \cdot 132,223 - (1,089)^2}{9 \cdot 8}} = 7.533$$
We move on to the test.
$H_0: \sigma = 29$
$H_1: \sigma < 29$

The test statistic is
$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{8 \cdot 7.533^2}{29^2} = 0.540$$
In a left-tailed test at the 0.01 significance level with $df = 8$, the critical value is $\chi^2 = 2.733$.
In the row for 8 degrees of freedom, the test statistic lies below 1.344, so the $P$-value is less than 0.005.
We reject the null hypothesis.
The sample data support the claim that the population standard deviation of supermodel body weights is less than 29lb.