AMS7: REVIEW PROBLEMS.
CHAPTER 3

EXAMPLES: Conditional probability, Bayes Theorem, Risk and odds
Wednesday April 22nd, 2015
CONDITIONAL PROBABILITIES

• Pregnancy test: 99 subjects randomly selected

<table>
<thead>
<tr>
<th></th>
<th>POSITIVE TEST RESULTS</th>
<th>NEGATIVE TEST RESULTS</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>PREGNANT</td>
<td>80</td>
<td>5</td>
<td>85</td>
</tr>
<tr>
<td>NOT PREGNANT</td>
<td>3</td>
<td>11</td>
<td>14</td>
</tr>
<tr>
<td>TOTAL</td>
<td>83</td>
<td>16</td>
<td>99</td>
</tr>
</tbody>
</table>

• Contingency table: Counts or frequencies for each category
• Row Totals: Total Pregnant: 85; Total not Pregnant: 14
• Column Totals: Total positive= 83; Total negative=16
Conditional Probabilities

- $P(\text{positive}|\text{pregnant})= \text{Pr. of getting someone tested positive given that the selected person was pregnant}= \frac{80}{85}=0.941$
- You can also use formula of Conditional Probability: $P(A|B)=\frac{P(A \ & \ B)}{P(B)}$
- $P(\text{positive}|\text{pregnant})=\frac{P(\text{pregnant} \ & \ \text{positive})}{P(\text{pregnant})}$
- $P(\text{pregnant} \ & \ \text{positive})=\frac{80}{99}$
- $P(\text{pregnant})=\frac{85}{99}$
- $P(\text{positive}|\text{pregnant})=\frac{\frac{80}{99}}{\frac{85}{99}}=0.941$

RESULTS ARE THE SAME!

Note that $P(\text{positive}|\text{pregnant})$ is not the same as $P(\text{pregnant}|\text{positive})$

$P(\text{pregnant}|\text{positive})=\frac{80}{83}=0.964$. $P(A|B) \neq P(B|A)$
Bayes Theorem

- In Orange County: 51% of adults are males (M); 9.5% of Males smoke cigars; 1.7% of females smoke cigars.
- A randomly selected subject was smoking cigar. What is the probability that the subject is male?

Notation: M=Males; C= cigar smoker
P(M)=0.51; P(non M (female))=0.49
P(C|M)=0.095
P(C|non M)=0.017
Bayes Formula

• \( P(M|C) = \frac{P(M) \times P(C|M)}{P(M) \times P(C|M) + P(M') \times P(C|M')} = \frac{0.51 \times 0.095}{0.51 \times 0.095 + 0.49 \times 0.017} = 0.853 \) (Rounded to 3 Decimals)

• **Interpretation**: Before the survey, probability of males is 51% (prior probability). After the survey, a randomly selected subject smokes cigar. There is an 83.5% probability that the surveyed subject is a male (posterior probability).
Risk and Odds

- Prospective study of Polio and Salk Vaccine

<table>
<thead>
<tr>
<th></th>
<th>Polio</th>
<th>No Polio</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Salk Vaccine</td>
<td>33 (a)</td>
<td>200,712</td>
<td>200,745</td>
</tr>
<tr>
<td>Placebo</td>
<td>115 (c)</td>
<td>201,114</td>
<td>201,229</td>
</tr>
</tbody>
</table>

- Risk of polio for children treated with Salk Vaccine vs. Risk of Polio for children given a Placebo
- \( P(\text{polio|Salk vaccine}) = \frac{a}{a+b} = \frac{33}{200,745} = 0.000164 \)
- \( P(\text{polio|Placebo}) = \frac{c}{c+d} = \frac{115}{201,229} = 0.000571 \)
Absolute risk reduction and relative risk

- **Absolute risk reduction:**
  \[ |P(\text{polio}|\text{Salk Vaccine}) - P(\text{polio}|\text{Placebo})| = |0.000164 - 0.000571| = 0.000407 \]

- **Relative risk:**
  \[ P_{\text{treatment}} / P_{\text{control}} = 0.000164 / 0.000571 = 0.287 \]

Interpretation: Polio rate for children given the Salk vaccine is 0.287 (≈1/3) of the polio rate for children given the placebo.

Considering the reciprocal: \(1/0.287 = 3.48\) (placebo groups are 3.48 times more likely to get polio)
**Number needed to treat**

- This is the number of subjects that must be treated in order to prevent one event (ex. A polio case).

Number needed to treat = \( \frac{1}{\text{absolute risk reduction}} \)

(rounded to the nearest integer) = \( \frac{1}{0.000407} = 2,457 \)

(rounded to the nearest integer).

**INTERPRETATION:** We need to vaccinate 2,457 children with the Salk vaccine to prevent one of the children from getting polio.
Odds Ratio

- Used in prospective or retrospective studies
- Example: 1:7 (1 to 7), 3:1 (3 to 1)
- Odds ratio = odds in favor of event for treatment/odds in favor of event for control group = $ad/bc$
- For Polio: Odds ratio = $33 * 201,114/115*200,712 = 0.287$
- In a prospective study we know beforehand the actual incidence rates. Use Relative Risk ratio
- In a retrospective study the incidence rate can be chosen by the researcher (Example: Choose a sample with 50% of incidence of hypertension). Use Odds ratio
- RELATIVE RISK SHOULD NOT BE USED WITH RETROSPECTIVE STUDIES.
- Prospective study: Use relative risk and/or odds ratio
- Retrospective study: Use odds ratio only
Mortality, Fertility and Morbidity

- **Crude mortality rate** in a particular year in the US = 
  \( \frac{\text{# of deaths}}{\text{total # in population}} \times k \) (\( k \) is a multiplier)

  Ex: 2,146 deaths in a population of 285,318,000 using \( k = 1,000 \):
  \( \frac{2,146}{285,318,000} \times 1000 = 8.5 \) (rounded).

  Interpretation: 8.5 people for each 1,000 people died

- **Crude fertility rate** =
  \( \frac{\text{# of births}}{\text{total # in population}} \times k \)

- **Morbidity (Disease) rate**:
  **Incidence rate** =
  \( \frac{\text{# reported cases of disease}}{\text{total number in population}} \times k \)