Review Quiz.

1) \[ \sum_{k=0}^{n} \frac{n!}{k!(n-k)!} a^k b^{n-k} = (a+b)^n \]
   \[ a = 7 \quad b = 3 \quad n = 6 \]
   \[ 10^n = 10^6 \]

2) \[ \frac{d}{dt} g(t) = \frac{1}{5} g(t) \] linear, homogeneous.

   \[ \frac{d}{dt} g(t) - \frac{1}{5} g(t) = 0 \]
   general solution: \[ C e^{t/5} \]
   arbitrary constant.

   \[ g(0) = 1 \]
   initial condition.

   \[ C e^0 = 1 \quad \Rightarrow \quad C = 1 \quad g(t) = e^{t/5} \]

   \[ g(t) > 4 \quad \Rightarrow \quad e^{t/5} > 4 \quad \Rightarrow \quad t > 5 \ln 4 = 10 \ln 2 \]

HW1. P. 9-10 1, 2, 4, 5, 10. HW due on Tuesday's classes.
      P. 23 1, 2, 4, 6. (Tu - Tu cycle)

Coin tossing.

\{ head \}
\{ tail \}

\[ \begin{array}{c} X \end{array} \]
\[ \begin{array}{c} 1 \quad 0 \end{array} \]

\( n \) tosses, \( S_n \): # of heads ( # of times that heads came up ).
\( j \)th toss, \( j = 1, 2, \ldots, n \).

\[ X_j = \begin{cases} 1 & \text{if head} \\ 0 & \text{if tail} \end{cases} \]

\[ S_n = \sum_{j=1}^{n} X_j \]

process: random experiment.

outcome not known in advance

do know set of all possible outcomes.
Ex. \( \{ 0 \ldots 0 \} \)  
\( \# \) of polls \( \in [1, 10] \).

10 controller

Sample space of random experiment: the set of all possible outcomes \( \mathcal{S} \).

\{ sample point \\
\{ elementary event .

Ex. \( \mathcal{S} = \{1, 2, 3\} \)

\( \mathcal{N} = \{ x : \Omega(x) \} \leftarrow \) all \( x \) such that some property \( \Omega(x) \) is true.

\( 10 \text{ms} \leq x \leq 25 \text{ms} \) for example.

Examples:

1. die tossing (single die).
   \( \mathcal{S} = \{1, 2, 3, 4, 5, 6\} \).

2. 2 fair dice
   \( \mathcal{N} = \{(1,1), \ldots, (6,6)\} \)
   36 sample points
   or \( \mathcal{S} = \{0, 1\} \)
   \( \nearrow \) same \( \uparrow \) same # of spots for each die.

3. \( \{ 0 \ldots 7 \} \) 7 devices.
   Sample point: \# of responding devices. 9
   \( \mathcal{S} = \{1, \ldots, 6, 7, 8\} \)
   \( \uparrow \) none responded.

4. measuring response time of search engine. (min. 1ms).
   (e.g. Google).
\[ N = \{ \text{real } t : t \geq 1 \text{ ms} \} \quad \text{"continuous"}
\]

6. toss fair coin until 1st head.

- \( n \): # of tosses. \( N = \{1, 2, 3, \ldots \} \)
  \[ = \{H, TH, TTH, \ldots\} \]

\{ finite \} \{ discrete: finite or countable \}
\{ infinite \} \{ continuous \}

**event**: subset of sample space, corresponding to a statement
whether it is true or false once we do an experiment.

**Ex.** everybody in CMPE107 will get A+ (true or false?)
event \( A \) occurs: observed outcome is element of \( A \). (EA)

**Examples:**

1. die tossing
   
   \[ A = \{2, 3, 5\} \iff \text{"rolling a prime #"} \]

2. 2 fair dice tossing
   
   \[ A = \{(5, 6), (6, 5)\} \iff \text{"rolling an eleven"} \]

3. \( 9 \ldots 9 \) polling 7 device
   
   \[ A = \{6, 7, 8\} \iff \text{"more than 5 polls answered"} \]
   
   ↑ none (has to poll all of them).
+ measured response time is between 25 and 35.
\[ A = \{ t : 25 \leq t \leq 35 \} \]

5. \[ A = \{ H, TH, TTH \} \iff \text{"not more than 3 tosses needed".} \] (until get a head.)

<table>
<thead>
<tr>
<th>Set operation</th>
<th>Probability statement</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A \cup B )</td>
<td>union ( A ) or ( B ) occurs.</td>
</tr>
<tr>
<td>( A \cap B )</td>
<td>intersection both ( A ) and ( B ) occur.</td>
</tr>
<tr>
<td>( \bar{A} )</td>
<td>complement ( A ) does not occur.</td>
</tr>
<tr>
<td>( A \cap B = \emptyset )</td>
<td>empty set ( A ) and ( B ) mutually exclusive</td>
</tr>
<tr>
<td>( \emptyset )</td>
<td>impossible event.</td>
</tr>
<tr>
<td>( A \subset B )</td>
<td>contained ( A ) implies ( B ).</td>
</tr>
</tbody>
</table>

Set operations and probability statements.

Venn diagram:

- \( \cap \rightarrow \begin{array}{c}
\text{A} \\
\bar{A}
\end{array} \) \( \cap \) \( A \) and \( B \) mutually exclusive.
- \( \bar{\cap} \) \( \cap \) \( \bar{\cap} \) \( A \) and \( B \) mutually exclusive.
- \( \cap \cap \cap \) \( A \) and \( B \) mutually exclusive.

\[ \bigcup_{n=1}^{\infty} A_n = A_1 \cup A_2 \cup A_3 \cup \ldots \]
\[ \bigcap_{n=1}^{\infty} A_n = A_1 \cap A_2 \cap A_3 \cap \ldots \]

Sequential sample spaces tree diagram.
How do you measure probability?

rolling an eleven \( \left( \frac{1}{36} \right) \)

A = \{(5,6), (6,5)\}