HW #2 Due next Tuesday
44 #1, 2
45 #14, 18
82 #2, 4
87 #1, 4, 8
97 #8, 12
106 #4, 9
119 #4, 8

Combinatorics

\[ P(E) = \frac{\text{# of favorable outcomes}}{\text{# of outcomes}} \]

Looking at the number of permutations of k objects out of n elements

\[ \begin{align*}
&\overset{n}{\uparrow} \quad \overset{n-1}{\uparrow} \quad \overset{n-2}{\uparrow} \quad \cdots \quad \overset{n-k+1}{\uparrow} \\
\text{ways to select 1st elt.} &\quad \text{ways to select 2nd object} &\quad \text{ways to select 3rd object} &\quad \text{ways to select n-k object}
\end{align*} \]

\[ n(n-1)(n-2) \cdots (n-k+1) = \frac{n!}{(n-k)!} \]  

# of ways to select k from n: where order matters
choose k objects from n objects without replacement

How many ways are there to arrange k elements? Answer: k!

# of combinations of k elements from n elements
\[ \frac{n!}{(n-k)! \times k!} \]
\[ = \binom{n}{k} \]

\[ = C(n, k) \]
Example.

5 devices which device requires service by the controller

\[ \Omega = \{ (x_1, \ldots, x_5) \mid \text{such that } x_i = \begin{cases} 0 & \text{not ready} \\ 1 & \text{ready} \end{cases} \} \]

Size of \( \Omega \) = \( |\Omega| = 2^5 \) = 2 states

Event = \( \{ \text{exactly 3 are ready} \} \)

\# of ways to select 3 from 5 = \( \binom{5}{3} = \frac{5!}{3!2!} = 10 \)

\[ \therefore P(\{E\}) = \frac{10}{32} \]

How many devices required so that exactly 3 devices are ready?

\( A_1 = \{ \text{1 device} \} \)

\( A_2 = \{ \text{2 devices} \} \)

\( A_3 = \{ \text{3 devices} \} \) i.e. choose other 2 ready devices

\[ \frac{4}{2} \]

\[ \therefore P(A_1) = \frac{6}{10} \]

Space is reduced to 10 pts

\[ A_2 : \begin{cases} \text{x}_1 = 0, \text{x}_2 = 1 \\ 0\text{thers} \in \binom{3}{2} \end{cases} \] \[ \therefore P(A_2) = \frac{3}{10} \]

\[ A_3 : \begin{cases} \text{x}_1 = 0, \text{x}_2 = 0, \text{x}_3 = 1 \\ 0\text{thers} \in \binom{2}{2} \end{cases} \] \[ \therefore P(A_3) = \frac{1}{10} \]

\[ \text{Conditional probability} \]

\[ \sum_{\{B \} \neq 0} \]

\[ P(A|B) = \frac{P(A \cap B)}{P(B)} \]

\[ A = \{ \text{live to be 100} \} \]

\[ B = \{ \text{male} \} \]

B is called "conditioning event"
Multiplication rule: \[ P(E \cap B) = P(E | B) P(B) \]

\[ P(A_1 \cap A_2 \cap \ldots \cap A_n) = P(A_1) P(A_2 | A_1) P(A_3 | A_1 \cap A_2) \ldots P(A_n | A_1 \cap \ldots \cap A_{n-1}) \]

\[ = P(A_1 \cap A_2) \times \ldots \times P(A_n | A_1 \cap \ldots \cap A_{n-1}) \]

Example:

100 installations in your region,

75% are customers

Visit 3 installations, P{3 installations are all customers}.

\[ P(A_1 \cap A_2 \cap A_3) = P(A_1) P(A_2 | A_1) P(A_3 | A_1 \cap A_2) \]

\[ = \frac{75}{100} \times \frac{74}{99} \times \frac{73}{98} \]

\[ \approx 0.42 \text{ low!} \]

Example:

\[ \Omega = \{(x_1, x_2, \ldots, x_5)\} \quad |\Omega| = 2^5 = 32 \]

3 ready - there are 10 points conditioning event

\[ P(3 \text{ ready}) = \frac{10}{32} \]

\( A_1 \) device needed

\[ P(A_1 \cap \text{3 ready}) = \frac{6}{32} \quad P(E \cap A_1, 3 \text{ ready}) = \frac{6/32}{10/32} \]
\[ P_{EB1B3} = \frac{P_{EB \cap B3}}{P_{EB3}} = 1. \]

**Example.**

- 5000 memory chips
- 4000 y 5% defective
- 1000 x 10% defective (from Supplier X)

<table>
<thead>
<tr>
<th>Label</th>
<th>Event</th>
<th>( P_{A3} )</th>
<th>( P_{EB3} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Brand X</td>
<td>( \frac{1000}{5000} ) = 0.20</td>
<td>( \frac{100 + 200}{5000} ) = 0.06</td>
</tr>
<tr>
<td>B</td>
<td>chip defective</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ P_{A \cap B3} = \frac{100}{5000} = 0.02 \]

\[ P_{EA1B3} = \frac{P_{EANB3}}{P_{EB3}} = \frac{P_{EANB3}}{P_{EB3}} \]

New idea:

2 events \( A, B \) on \( \Omega \).

What is \( P_{EA1B3} \)?

- \( P_{EANB3} \)
- \( P_{EB3} \)
- \( P_{EANB3} \)
- \( P_{EB3} \)
- \( P_{EANB3} \)
- \( P_{EB3} \)

4 mutually exclusive pieces: \( A_1, \ldots, A_4 \) and

\( \cap A_i \cap A_j = \emptyset \) if \( i \neq j \)

Law of total probability:

\[ P_{EA3} = P_{EANA_1} + \ldots + P_{EANA_4} \]

\[ = P_{EAA_1} P_{EAA_1} + \ldots + P_{EAA_4} P_{EAA_4} \]

\( \emptyset \cup = UA_i \) "collectively exhaustive" \( \Rightarrow \) partition of \( \Omega \)

for \( n \) pieces conditioned on \( A \).