HW #1  Due Tues Jan 24

pages  problem #
9-11   2, 4, 6, 10, 18
23-24  2, 4, 6, 12, 16
44-45  1, 2, 14, 18

Divide errors into 3 categories: syntax, I/O, other
sample of 100 programs
20 have syntax errors "S"
10    I/O    "I"
5     other  "O"
6    both syntax and I/O
2    I/O and other
3    syntax and other
1    3 types

What is probability that a program has a syntax error
or I/O or both?
1. \( P\{\text{syntax or I/O or both}\} = ? \)
2. \( P\{\text{error}\} = ? \)

Event \( S \) I O S\(\cap\) I S\(\cap\) O I\(\cap\) O S\(\cap\) I\(\cap\) O

probability \( \frac{20}{100} \) \( \frac{10}{100} \) \( \frac{5}{100} \) \( \frac{6}{100} \) \( \frac{3}{100} \) \( \frac{2}{100} \) \( \frac{1}{100} \)

\( P\{S\cap I\} = P\{S\} + P\{I\} - P\{S\cap I\} \)
\[ = \frac{20}{100} + \frac{10}{100} - \frac{6}{100} = \frac{24}{100} = \frac{6}{25} \]

\[ \text{if didn't subtract would count overlap twice} \]
\[
P_{\text{error}}^3 = P\{S \cup I \cup O\}^3 \\
= P\{S \cup (I \cup O)\}^3 \\
= P\{S\}^3 + P\{I \cup O\}^3 - P\{S \cap (I \cup O)\}^3
\]

\[
= P\{S\}^3 + P\{I\}^3 + P\{O\}^3 - P\{I \cap O\}^3 - P\{(S \cap I) \cup (S \cap O)\}^3 \quad \text{by distributive law (\#)}
\]

Note that (\#) = \(P\{S \cap I\}^3 + P\{S \cap O\}^3 - P\{S \cap I \cap O\}^3\)

\[
= P\{S\}^3 + P\{I\}^3 + P\{O\}^3 - P\{I \cap O\}^3 - P\{S\}^3 P\{I\}^3 - P\{S\}^3 P\{O\}^3 + P\{S\} P\{I\} P\{O\}.
\]

\[
= \frac{25}{100} = \frac{1}{4}.
\]

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**Combinatorics**

Two ways of selecting a sample:

1) with replacement

2) without replacement

Permutation of order \(k\) = ordered selection of \(k\) elements

Combination of \(k\) = unordered selection (of \(k\) elements)

\[\{x, y, z\} \quad \text{Draw a sample of 2:}\]

9 permutations with replacement: \(9 \Rightarrow xy, xz, zx, yx, yz, zy, zx, zy, xz\)

6 permutations without replacement: \(xy, xz, yx, yz, zx, zy\)

3 combinations \(\text{w/o replacement: } xy, xz, yz\)

Imagine I have \(k\) objects: How many ways can I place them? (to arrange them?)
A multiplication principle \( nm \)

B \( m \)

k elements \( k! \)

(Note: \( 0! = 1 \))

Select k out of n elements w/o replacement

How many permutations are there?

\[
\begin{vmatrix}
\begin{array}{ccccc}
 n & n-1 & n-2 & \cdots & n-k+1 \\
 1^{st} \text{element} & 2^{nd} \text{element} & \text{k}^{th} \text{element} \\
\end{array}
\end{vmatrix} = \frac{n!}{(n-k)!}
\]

Note: \( \frac{n(n-1)(n-2)(n-k+1)(n-k)\cdots2\cdot1}{(n-k)\cdots2\cdot1} = \)

How many combinations? \( \frac{n!}{(n-k)!k!} \)

k! ways of arranging k elements so since it's a combination we have to divide \( \frac{n!}{(n-k)!} \) by k!

w/replacement: \( n \cdot n \cdots n = n^k \)

Example. Imagine 5 devices: \( \square \square \square \square \)

\( \bigcap \text{b} = \{ x_1, x_2, x_3, x_4, x_5 \}^2 \)

\( \cap \text{b} = \{ 0 \text{ not ready} \}
\]

\( x_i = \begin{cases} 0 & \text{not ready} \\ 1 & \text{ready} \end{cases} \quad i=1, \ldots, 5 \)
What is the total number of sample points?
2 states 5 devices \( = 2^5 = 32 \).

\[
\text{Event} = \{ \text{exactly 3 devices are ready} \} \\
\text{Size of event} = \frac{5}{3!2!} = \binom{5}{3} = 10 \\
\text{.. Probability of event} \frac{10}{32} \\
\text{it's a combination w/o replacement} \\
\text{(order doesn't matter; positions only) imagine instead color. which color?}
\]

Knowing that exactly 3 are ready, how many polls are needed to find the first ready? It could take 1, 2 or 3 polls. Call these events \( A_1, A_2, A_3 \).

\[
\text{Size of } A_1 = |A_1| = \binom{4}{2} \\
|A_1| = 6 \\
\text{Prob } \{ A_1 \} = \frac{6}{10} \text{ knowing 3 are ready.}
\]

\[
\text{Size of } A_2 = |A_2| = \binom{3}{2} \\
|A_2| = 3 \\
\text{Prob } \{ A_2 \} = \frac{3}{10}
\]

\[
|A_3| = \binom{2}{2} = 1 \\
\text{Prob } \{ A_3 \} = \frac{1}{10}
\]

~ Conditional Probability ~

What is probability of A knowing that B occurred \( \uparrow \) conditioning event

\[
= \text{prob of } A \text{ given } B
\]
\[ P[A \mid B] = \frac{P[A \cap B]}{P[B]} \text{ if } P[B] \neq 0 \]

\[ \Omega = \{A, B, \ldots\} \]

\[ P[B] \cap B = \frac{P[B \cap B]}{P[B]} = 1. \]

**Multiplication rule:**

\[ P[A \cap B] = P[A] P[B \mid A] \text{, if } P[A] \neq 0 \]

\[ = P[B] P[A \mid B] \text{, if } P[B] \neq 0 \]

\[ \text{probability of } A \text{ and } B \text{ occurring jointly} \]

\[ \text{Suppose } A_1, \ldots, A_n \]

\[ P[A_1 \cap A_2 \cap \ldots \cap A_n] = ? \]

\[ = P[A_1] P[A_2 \mid A_1] P[A_3 \mid A_1, A_2] \ldots P[A_n \mid A_1, \ldots, A_{n-1}] \]

\[ \text{Comma means } A_1 \cap A_2 \]

\[ = P[A_1 \cap A_2] P[A_1 \cap A_2 \cap A_3] \ldots P[A_1 \cap A_2 \cap \ldots \cap A_n] \]

**Example:** 7.5% of installations already have your software.

Visit 3 different installations/sites.

\[ P\{3 \text{ random sites knowing } 7.5\% \text{ are customers}\} = ? \]

\[ P\{\text{first site is customer}\} = \frac{75}{100} \]

\[ P\{E_1\} = \frac{75}{100} \]

\[ E_1 \cap E_2 \text{ second site} \]

\[ P\{E_1 \cap E_2\} = P\{E_1\} P\{E_2 \mid E_1\} \]

\[ = \frac{75}{100} \times \frac{74}{99} \]

\[ Etc. \text{ for all 3 sites} = \frac{75}{100} \times \frac{74}{99} \times \frac{73}{98} = 0.418 \]

**Example:**

5000 chips, 4000 chips have 5% are defective.

From X, 1000 chips, 10% are defective.

\[ P\{A\} \text{ defective in } X = \frac{1000}{5000} = 0.2 \]

\[ P\{B\} \text{ defective in } X = \frac{300}{5000} \]

\[ P\{A \cap B\} \text{ defective in } X = 0.2 \times 0.2 = ? \]