CMPS130 F2011: Final Exam

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All problems are 10 points unless otherwise marked. There are 328 points possible.

1. Construct a DFA over the alphabet 0,1 that accepts exactly those strings in which the first 2 input symbols are the same or the last 2 input symbols are the same but not both. (no string of less than 3 symbols should be accepted)

2. Construct an NFA or DFA over a,b which accepts all strings but the empty string.

3. Which one of the following is true?
   a. All 1PDAs are PDAs
   b. All PDAs are 1PDAs
   c. All DPDAs are PDAs
   d. All NFAs are DFAs.

4. Prove or disprove: For each alphabet Sigma, Square(sigma) = “set of strings in Sigma* with length = size (number of symbols) of Sigma squared” is context free. (For example, if Sigma is a,b,c, Square(sigma) is all strings of a,b,c with length 9.)
5. Construct a NFA $M$ over the alphabet 0,1 that accepts exactly those strings which contain an odd number of consecutive 1s as a substring. (use some non-determinism!)

6. Give a regular expression for the language accepted by the machine $M$ above.

7. Using the subset construction, produce a DFA $M'$ that accepts exactly the same language as $M$.

8. USING THE PRODUCT CONSTRUCTION (from two simple DFAs) construct a DFA that accepts the language $L$ made up of strings of $a,b$ that begin with $a$ and end with $b$. 
9. Consider the regular language L:

L={x in \{a,b\}* | x contains at least one 2-symbol palindrome but no 3-symbol palindromes as a substring}

(a) (15 points) Give a MINIMUM SIZE DFA for the language L!
(b) (5 points) Give a regular expression for L.
10. (3 points each part) For each of the following languages state whether they are regular or not by CIRCLING the letters of the regular languages! No need to justify your answer! But show use of the Pumping Lemma on any one uncircled language to justify your answer. (15 points)

NOTE: $x^y$ means x to the power of y.

a. The language generated by the regular expression $cab^*adfj(kd)^*kj$.

b. The intersection of 79 regular languages.

c. Strings whose length is divisible by 7.

d. $\{a^{2i}b^{2j} | i \text{ is odd and } j > i\}$

e. The set of primes greater than $1000000000000$.

f. The empty set intersection $\{a\}$.

g. $\{x \in \{a,b\}^* | \text{number of } a \text{ is not divisible by } 79\}$

h. $\{x \in \{a,b\}^* | \text{number of } a \leq \text{number of } b, \text{number of } b > 8\}$

i. A language not accepted by any Non-deterministic Finite Automaton.

j. $\{x \in a^* | \text{Number of } a > \text{Number of } b. \}$

k. Palindromes of $\{a,b\}$ that are of length 79.

l. $A^*$, where $A$ is empty.

m. The set of odd length strings of $\{0,1\}$ with no middle symbol.

n. The set of prime integers less than $100203034040506040470707805$.

o. The set of strings of $\{a,b\}$ ending
with a palindrome of size 110101016.

p. The set of strings of \{a,b\} in which the number of as and the number of bs are each NOT divisible by 3731020032030400500600407.

q. The set of strings in which the number of 0s is not a prime number.

r. A language which has a string of length 1007, and no string longer than 1550505002 symbols.

s. The intersection of 177132487478 regular languages.

t. The set of strings of even # of parentheses.
11. (25 total points)

(a) Consider the language:

\[ L = b^n b^{2n}, \text{"read as n bs, followed by 2n bs"}. \]

Is this language regular? If yes, give a regular expression for \( L \). Else continue.

(b) Is this language context free?

(c) If the language is not context free, show why not using the Pumping Lemma for CFLs, else do the remaining parts:

(d) Prove not regular using the pumping lemma.

(e) Give a CFG for \( L \).

(f) Give a grammar in Chomsky Normal Form for \( L \).

(g) Show using CYK algorithm whether bab is in the language.

(h) Give a 1PDA for \( L \).
12. (25 total points)

(a) Consider the language:

Odd length strings of a, b, c whose middle, beginning and end are the same symbol (the language includes all 1 symbol strings, but does not include epsilon)

Is this language regular? If yes, give a regular expression for L. Else continue.

(b) Is this language context free?

(c) If the language is not context free, show why not using the Pumping Lemma for CFLs, else do the remaining parts:

(d) Prove not regular using the pumping lemma.

(e) Give a CFG for L.

(f) Give a grammar in Chomsky Normal Form for L.

(g) Show using CYK algorithm whether bab is in the language.

(h) Convert L into a 1PDA.
13. (25 total points)

(a) Consider the language:

\[ \text{even length palindromes on \{a,b,c\} which only} \]
\[ \text{use one symbol (possibly repeated).} \]

Is this language regular? If so, give a regular expression for it. Else, continue...

(b) Is this language context free?

(c) If the language is not context free, show why not using the Pumping Lemma for CFLs, else do the remaining parts:

(d) Prove not regular using the pumping lemma.

(e) Give a CFG for L.

(f) Give a grammar in Chomsky Normal Form for L.

(g) Show using CYK algorithm whether bab is in the language.

(h) Convert L from CNF into a 1PDA.
14. (3 points each) Please answer the following questions TRUE OR FALSE - no need to justify your answers.

(a) If problem A reduces to problem B, then B is at least as hard to solve as A.
(b) All languages are r.e. or recursive.
(c) Every language is either infinite or has a DFA.
(d) 1PDAs and NPDAs accept the regular language (and possibly others).
(e) DPDAs and 1PDAs accept the regular languages (and possibly others).
(f) The intersection of a Regular Language with a Regular Language is always recursive.
(g) The intersection of a Context Free Language with a Context Free Language is always Context Free.
(h) The intersection of 17 r.e. languages is always context free or infinite.
(i) All empty sets are finite.
(j) All finite sets are r.e.
(k) The set of palindromes over a,b is Context Free but is not acceptable by a DPDA.
(l) Every 1PDA can be converted into a grammar in Chomsky Normal Form.
(m) The intersection of two context free languages is never equal to the union of two regular languages.
(n) The empty set is acceptable by NFA but not TM.
(o) The complement of a context free language is always acceptable by PDA.
(p) The complement of a regular language is always infinite or deterministic context free.
(q) All Turing machines halt on some inputs.
(r) There is no Turing machine that can recognize whether its input is a prime number.

(s) NonDeterministic Turing machines are less powerful than Turing Machines with 100000 tapes.

(t) If a Turing machine accepts baaaaaaaaaab it must halt.

(u) Whether a Turing machine goes into an infinite loop (doesn’t halt) on bab can always be determined without simulating the machine.

(v) All languages are r.e.

(w) Whether Turing Machine M halts on string w is r.e. but not recursive.

(x) All regular languages are r.e.

(y) There are more possible languages than Turing machines.

(z) There is a Turing machine that accepts a not r.e language.