CMPS272: Homework #2

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Due Date: Th Jan 26, 2012, beg. of class

Part I (handed in on paper at beginning of class)

I-1) For the general 2x2 matrix games presented in section (1.2), what can you say about behavior when \( w_1 = w_2 = 0 \)? How do you interpret this situation? What can you say about behavior when \( w_1 > w_2 = 0 \)? How do you interpret this situation? What can you say about behavior when \( w_1 < w_2 = 0 \)? How do you interpret this situation? Remark. In a later chapter, we will discuss extensive form games. They often give rise to situations in which two or more \( w_{ij} \)'s are equal. Therefore, when the payoff table comes from an extensive form game, it is much more likely to have some \( w_i = 0 \).

I-2 A \( t_1 \) replicator step from time \( t \) is \( s_i(t + t_1) = \frac{s_i(t) W_i}{\sum_{j=1}^{n} s_j(t) W_j} \). Show that a \( t_1 \) step from \( t \) followed by a \( t_2 \) step is the same as a \( t_1 + t_2 \) step from \( t \).

I-3 Show that the discrete replicator solution

\[
s_i(t) = \frac{s_i(0) e^{t(w_i - w_n)}}{\sum_{j=1}^{n-1} s_j(0) e^{t(w_j - w_n)} + 1 - \sum_{j=1}^{n-1} s_j(0)}
\]

satisfies

\[
\ln \left( \frac{s_i(t)}{1 - \sum_{j=1}^{n-1} s_j(t)} \right) - \ln \left( \frac{s_i(t-1)}{1 - \sum_{j=1}^{n-1} s_j(t-1)} \right) = w_i - w_n.
\]
and more generally

\[
\ln \left( \frac{s_i(t)}{s_j(t)} \right) - \ln \left( \frac{s_i(t-1)}{s_j(t-1)} \right) = w_i - w_j.
\]

See slide 18 of Manfred’s Jan 19 lecture. You can use any partial step that you find on the slides. Just reference the slide number.
I-4 Show that the discrete replicator solution \( s_i(t) = s_i(t-1)e^{w_i} \sum_{j=1}^n s_j(t-1)e^{w_j} \) does not satisfy

\[
\ln s_i(t) - \ln s_i(t-1) = w_i - \sum_{j=1}^n w_j s_j(t)
\]

by giving an expression for the lhs. For a discussion see slide 18.

Part II. Spreadsheet - submit to rps2bdm@gmail.com

II-1) Build a spreadsheet to simulate discrete replicator dynamics for general HD games. Have labelled cells near the top for entering the parameters \( v \) and \( c \), and write formulas that display the resulting 2x2 matrix. Include another labelled cell for the initial H share. Then build up a row and column display similar to that used in the chapter 1 spreadsheet to track H and D shares for 30 or more periods. Verify that you get convergence to the interior steady state \( v/c \) for appropriate values of the parameters.

II-2 Build a similar spreadsheet to simulate discrete replicator dynamics for RPS and other 3x3 matrix games. Check the claims made in section ?? about stability and spiraling.

Part III. Prospectus for term project – submit one copy per group, in class.

III) In a page or so, name your research project, say who is involved (and their status, eg. undergrad cmps, or phd bio), formulate an answerable research question, and say something about intended methods and relevant published research.