1. When $w_i = w_j = 0$, $\Delta w$ is always 0, and any state is a steady one. 
   When $w_i > w_j = 0$, $\Delta w$ is always positive, except for $S_0 = 1$. So $S_0 = 1$ is the steady state. 
   When $w_i < w_j = 0$, $\Delta w$ is always negative except for $S_0 = 0$. So $S_0 = 0$ is the steady state.

2. A $t_1$ step followed by a $t_2$ step so,

\[
S_i(t + t_1 + t_2) = \frac{S_i(t) w_i^{t_1} w_j^{t_2}}{\sum_j S_j(t) w_j^{t_1}} \frac{S_j(t) w_i^{t_1} w_j^{t_2}}{\sum_j S_j(t) w_j^{t_1}} \frac{S_i(t) w_i^{t_1} w_j^{t_2}}{\sum_j S_j(t) w_j^{t_1}} = \frac{S_i(t) w_i^{t_1} w_j^{t_2}}{\sum_j S_j(t) w_j^{t_1}} = S_i(t + t_1 + t_2)
\]

3. 
\[
\ln \frac{S_i(t)}{S_j(t)} = \ln \frac{S_i(0) e^{t(W_i - W_j)}}{S_j(0) e^{t(W_i - W_j)}} = \ln \frac{S_i(0)}{S_j(0)} + t(W_i - W_j)
\]

\[
\ln \frac{S_i(t-1)}{S_j(t-1)} = \ln \frac{S_i(0)}{S_j(0)} + (t-1)(W_i - W_j)
\]

\[
\ln \frac{S_i(t)}{S_j(t)} - \ln \frac{S_i(t-1)}{S_j(t-1)} < W_i - W_j
\]

4. 
\[
\ln \frac{S_i(t)}{S_i(t-1)} = \ln \frac{S_i(t+1) e^{W_i}}{S_i(t) \sum_j S_j(t+1) e^{W_j}} = W_i - \left( \ln \sum_j e^{W_j S_j(t+1)} \right)
\]