Adaptive Business Cycles

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Introduction

The most interesting features of evolutionary games, besides the ability to select among multiple Nash equilibria, are their dynamic properties. These dynamic properties are best displayed in a stochastic setting. In the usual deterministic setting, dynamics disappear in the long run (or they are stable orbits that repeat themselves). Studying evolutionary dynamics in a stochastic setting has a potential economic application: the modeling of business cycles.

This paper is organized in the following way. Part 1 summarizes the standard economic analysis of business cycles. Part 2 presents a brief review of the adaptive business cycle literature. Part 3 briefly discusses Minsky’s theory of financial instability, that we try to model in what follows. In parts 4 to 8 we study a formal evolutionary model of liquidity management. Part 8 illustrates some simulations of the model obtained under alternative calibrations and concludes. In the appendix we further discuss the standard economic approach to modeling business cycles. We also discuss alternative strategies to model business cycles using evolutionary game theory.

1 Business Cycles

Economic activity is characterized by cyclical fluctuations over time, especially at the aggregate level and in financial markets. The following picture illustrates the typical pattern of detrended GDP and aggregate investment.
The standard methodology adopted in modeling business cycles involves four steps:

1. specifying a model of the economic variables based on the optimal choice of agents;
2. introducing exogenous random shocks to some parameters of the model;
3. calibrating the steady state parameters of the model and those of the stochastic processes;
4. studying the dynamics through simulations.

In this framework business fluctuations are interpreted as optimal responses to exogenous variations in some parameters of the economy. Modeling business cycles using an evolutionary (or adaptive) framework provides an alternative interpretation. In an evolutionary game agents do not necessarily choose the optimal strategy. Instead they can learn from past observations. In this learning process agents can overshoot or undershoot their goals. The basic intuition we want to explore in this paper is that the dynamics of the learning process may be the cause of cycles as seen in economic data.

2 Review of the Literature

The idea of modeling business cycles using adaptive dynamics dates at least back to Goodwin (1951). This earlier models tended to focus on the economy as a whole instead of modeling
the choice agents. The use of explicit microfoundations is characteristic of more recent adapt-
tive business cycle literature. A number of recent contributions have focused on adaptative
behavior in financial markets, as summarized in the review of Hommes (2006). For instance,
Brock and Hommes (1998) explore the dynamics of an asset pricing model with heteroge-
neous beliefs, where agents choose from a set of predictors of future prices of a risky asset.
Agents are assumed to revise their beliefs in each period according to a fitness measure. Asset
price fluctuations are thus driven by evolutionary dynamics. Friedman and Abraham (2008)
study how bubbles and crashes can emerge in asset markets. In their model a population of
fund managers choose the leverage of their portfolios. Leverage decisions are adjusted over
time based on gradient dynamics. The model generates bubble and crashes when it includes
an endogenous market risk premium and constant gain learning. Constant gain learning is
a form of learning that assigns a constant weight to the most recent observations. The
assumption of constant gain learning is motivated by the observation that "investors seem to
judge managers by the overall historical track record, with greater emphasis on more recent
results" (Friedman and Abraham 2008, p. 929). In our model we employ a similar form of
short memory learning. Other contributions study evolutionary dynamics of business cycles
generated by firms’ investment decisions. For instance Dosi et al. (2006, 2008) present evolution-
ary models of industry dynamics yielding endogenous business cycles. In particular their
result is due to the assumption of boundedly rational expectation formation.

3 Financial Instability

We try to apply evolutionary dynamics to a model of business cycles in financial markets. We
use a model inspired by Minsky’s theory of financial instability. According to Minsky (1986,
p.210), "experience indicates that our economy oscillates between robust and fragile financial
structures and financial crises require the prior existence of a fragile financial structure. We
need to explain how fragility emerges and how robust situations are reconstituted" (p.210).

In Minsky’s theory there is a large population of financial units engaged in different strate-
gies in terms of portfolio liquidity. In Minsky’s terminology speculative finance is any scheme
that involves financing of long-term, illiquid assets by short-term borrowing. In his theory
the state of the population determines the system’s financial fragility: "the mixture of hedge,

\footnote{We would like to thank David Florian Hoyle for bringing these two papers to our attention.}
speculative, and Ponzi finance in an economy is a major determinant of its stability. The existence of a large component of positions financed in a speculative or a Ponzi manner is necessary for financial instability. A question that needs to be addressed is, What determines the changing proportions of units in each financing mode?" (p.209). Replicator dynamics, would be our answer.

Minsky argued that starting from an initial condition where holding very liquid assets is the most common strategy; it is profitable to shift to less liquid portfolios: "in a system dominated by hedge finance the pattern of interest rates, short term rates being significantly lower than long term rates, are such that profits can be made by intruding speculative arrangements" (p.210). Minsky would argue that the thrust towards speculative finance will not fade out gradually, resulting instead in a self-sustaining bubble. However, at some point, when the most common strategy is holding very illiquid, risky portfolios and liquidity shortages are widespread it will be best to hold more liquidity. Thus, "the economy emerges from a recession that follows a financial crisis with a more robust structure than it had when the crisis took place" (p.210), but this is just the beginning of a new cycle.

4 A Simple Model of Liquidity Management

In what follows we use a formal model of bank reserve management. Before working out the details of the model, we want to stress that this is just an example that could be generalized to other types of firms and financial arrangements. The general problem we want to focus on is the short-term financing of illiquid assets, which for a bank coincides with the problem of allocating reserves and loans out of deposits. While a liquidity shock in the following model is interpreted as a bank run, this is only an extreme example of a situation in which short-term debt is not renewed by the lender.

We use a model of reserve management taken from Freixas and Rochet 1999 (p. 228). A bank wants to determine the quantity \( R \) of liquid reserves to be held, out of a total amount \( D_{ep} \) of deposits. The remaining \( (D_{ep} - R) \) is invested in illiquid loans. Reserves yield an interest rate \( r \) and loans yield \( r_L \) and \( r_L > r \). (Note also that in this model the bank does not pay interest on the deposits). The amount of withdrawals at the end of the period is a random variable \( z \). If the realization of \( z \) is greater than \( R \) the bank has a liquidity shortage and it has to pay a penalty \( k (z - R) \) (there are a number of possible explanations for the
nature of this cost, see Freixas and Rochet 1999, p. 228).

\[
\pi = r_L (Dep - R) + rR - kE [\max (0, z - R)]
\]

We simplify the model in the following ways:

- we assume reserves do not yield any interest: \( r = 0 \);
- we adopt the following normalization: \( Dep = 1, R \in [0; 1] \) and \( L = 1 - R \);
- we allow only two discrete strategies: high reserves, \( R_h \), or low reserves, \( R_l \) (with \( R_h > R_l \)).

The payoff function becomes:

\[
\pi_{i \in \{h, l\}} = r_L (1 - R_i) - kE [\max (0, z - R_i)]
\]

In our model there is a large population of banks and we denote by \( p_h \) and \( p_l \) the shares of the two strategies in the population. In what follows we introduce three major changes to the model: 1) we make the interest rate depend on the population state (in an alternative specification we make also the expected cost of liquidity shortage endogenous to the state), 2) we substitute rational expectations with a form of belief learning and 3) we use a logit decision rule.

5 Determination of the Interest Rate

For a bank the issue of a loan is equivalent to a purchase of an illiquid asset that yields \( r_L \).

We assume a standard specification for the asset market:

\[
\text{asset supply: } S = P^\alpha, \alpha > 0
\]

\[
\text{market clearing: } S = D
\]

\[
\text{interest rate equation: } P = \frac{1}{1 + r_L}
\]

which yields:
\[ r_L = \frac{1}{D^2} - 1 \]

which is decreasing in aggregate asset demand.

Aggregate asset demand is determined by the sum of loan issues \((L)\) over the two strategies weighted by the respective frequencies in the population:

\[ D = \sum_{i \in \{h, l\}} p_i L_i = \sum_{i \in \{h, l\}} p_i (1 - R_i) \]

Note that \(D\) is also a measure of the overall illiquidity of the financial system.

6 The Distribution of Liquidity Shocks

To simplify we assume that the liquidity shock \(z\) is bimodal: there can be either no shock or a total withdrawal of the deposits:

\[ z \in \{0, 1\} \]

Also we assume each bank is hit by the same shock. As a baseline model we assume that \(P(z = 1)\) is exogenous (set to 0.5 in the first simulation). Then, as an alternative specification, we will assume that the probability of a withdrawal depends on the state of the population in terms of liquidity. The intuition is that the more the illiquid the bank system is the more likely depositors are to withdraw funds out of fear of a credit crunch. We do not provide a full explanation for this assumption. In a more complete model, the liquidity shock should be modeled as the outcome of a Diamond and Dybvig (1983) game, where depositors decide whether to withdraw or not. It is reasonable to assume that with imperfect information the probability of a withdrawal is increasing in the actual exposure of the bank system to an illiquid position. Thus we want \(P(z = 1)\) to be increasing in the overall illiquidity of the population \(D\). We will assume the following relation:

\[ P(z_t = 1) = D^\rho \]

with \(\rho > 0\) (n.b.: \(D^\rho \in [0; 1]\) since \(D \in [0; 1]\)).
7 Belief Learning

At the beginning of period $t$ a bank can observe the current interest rate but does not know if there will be a liquidity shock or not. Thus the bank has to form expectations on $E_t [\max (0, z_t - R_t)]$, which reduces to the estimation of $E_t (z_t)$. We assume the bank estimates this term using a form of backward-looking belief learning:

$$\tilde{E}_t (z_t) = \frac{1}{\sum_{\tau=0}^{t-1} \gamma^\tau} \sum_{\tau=0}^{t-1} \gamma^\tau z_{t-\tau}$$

At the end of the current period the liquidity shock is realized and beliefs are updated in the beginning of the next period.

8 Logit Decision Rule

To sum up, the expected payoff to a unit playing strategy $i \in \{h, l\}$ in period $t$ is:

$$\pi_{it} = r_{Lt} (1 - R_{it}) - k \max \left(0, \tilde{E}_t (z_t) - R_{it}\right)$$

where

$$r_{Lt} = \frac{1}{D_t^2} - 1$$

$$\tilde{E}_t (z_t) = \frac{1}{\sum_{\tau=0}^{t-1} \gamma^\tau} \sum_{\tau=0}^{t-1} \gamma^\tau z_{t-\tau}$$

$$D_t = \sum_{i \in \{h, l\}} p_{it} (1 - R_{it})$$

We determine strategy shares using a logit decision rule:

$$p_{it+1} = \frac{\Pi_{it}}{\sum_{j \in \{h, l\}} \Pi_{jt}}$$

where
\[ \Pi_t = e^{\beta \tau_t} \]

9 Simulation Results

The parameters used in the baseline simulation are: \( R_h = 0.9, \ R_i = 0.1, \ \alpha = 1, \ k = 4, \ \beta = 1, \ P(z = 1) = 0.5 \). The results are robust to different choices of \( \alpha \) and \( k \). The choice of \( \beta \) is motivated by the property that a too high or too low value of the decisiveness coefficient would result in degenerate cases. The value of \( \gamma \) (memory) has a major role in the behavior of the model and this is explored in the first couple of simulations.

Before moving on to the simulation results, it can be useful to derive a benchmark for the evolutionary model. The evolutionary dynamics of the model can be contrasted to the rational expectation Bayesian Nash equilibrium of the game. If the probability distribution of the liquidity shock is known by the agents, the expected cost of a liquidity shortage is constant and does not vary with time. In other words the NE does not produce cycles. In our baseline model the Nash equilibrium share of each strategy is 0.5. ( derivation of this result is not reproduced because it is tedious).
First we simulate the model with perfect memory ($\gamma = 1$). Figure 2 shows the time series for the share of low reserve strategy in the baseline calibration with $\gamma = 1$. After an initial period of adjustment, over 700 simulation periods the series becomes relatively stable around 0.4.

Figure 2: Low Reserve Strategy Share Series - $\gamma = 1$
Reducing the memory coefficient raises the volatility of the series. We choose an intermediate value for the baseline model: $\gamma = 0.5$. Figure 3 reproduces the time series for the share of the low reserves strategy in this calibration.

![Share of Low Reserve Strategy](image_url)

Figure 3: Low Reserve Strategy Share Series - $\gamma = 0.5$

The series of Figure 3 displays wide fluctuations, ranging from periods where 90% of banks are liquid to periods where only 10% are liquid. Note that cycles do not fade out with time. This result replicates the basic behavior of a system oscillating between robust and fragile financial structures. Only with short memory ($\gamma < 1$) our model displays fluctuations in the long run. The assumption of short memory reflects the principle that “[recent] success breeds a disregard for the possibility of failure” (Minsky 1977, p. 146). Similarly Leijonhufvud argues that “transactors who have once suffered through a displacement of unanticipated
magnitude (on the order of the Great Depression, say) will be encouraged to maintain larger buffers thereafter – until the memory dims” (Leijonhufvud 1981, pp. 124, 125). Minsky provides an example of this same mechanism. He argued that previous experiences of default and crisis were neglected in the US postwar boom because “institutions, usages, and personnel changed between the financial trauma of the 1930s and the 1960s” (Minsky 1977, p. 146). According to this view short memory in firms and organizations is due to high turnover of practices and rules and broadly speaking to institutional change.
Next we provide some figures that mimic more closely the dynamics of business cycles. We focus on a shorter time horizon than that used in the previous figures (50 periods instead of 700). We also choose an ad hoc calibration that generates qualitatively interesting cycles: $R_h = 0.9$, $R_l = 0.1$, $\alpha = 0.7$, $k = 4$, $\beta = 1$, $P(z = 1) = 0.5$. Again a key element is short memory $\gamma = 0.7$. Figure 4 shows the series for the share of the high reserves strategy in this calibration. Figure 5 below illustrates the behavior of the rate of return on loans over time. Fluctuations in the strategy shares are reflected in fluctuations of the rate of return. Low rates are associated with booms, while interest rate spikes reflect falls in asset prices and credit crunches.

![Low Reserve Strategy Share Series](image)

Figure 4: Low Reserve Strategy Share Series
Figure 5: Interest Rate Series
Then it may be interesting to study the dynamics of the model when the liquidity shock distribution is made endogenous: \( P(z = 1) = D^\rho \). Some interesting behavior occurs for a relatively high value of the elasticity \( \rho \). Figures 6 and 7 illustrate the behavior of the model with \( \rho = 1.5 \). In this case the probability of a liquidity shock is in a population where only the high reserve strategy is played is just 3.16\%, which may be more realistic than the fixed 50\% probability assumed earlier.

Figure 6: Low Reserve Strategy Share Series - Endogenous Shock Probability
In conclusion the simulations show that with a sufficiently low $\gamma$ our the model displays wide fluctuations. Appropriate choices of the other parameters assist in generating cycles. However, the key element of adaptive business cycles in our model is short memory in belief learning of an stochastic process.
Appendix

a. Business Cycle Theory

As discussed above, the standard methodology adopted in modeling business cycles involves four steps:

1. specifying a model of the economic variables based on the optimal choice of agents;
2. introducing exogenous random shocks to some parameters of the model;
3. calibrating the steady state parameters of the model and those of the stochastic processes;
4. studying the dynamics through simulations.

Note that in this approach the exogenous stochastic shocks are not explained. Thus it is desirable for a model to be able to match the data starting from very general assumptions on the nature of the shocks. Ideally the shock should be thought of as a noise, a process with relatively small variance and little serial correlation. On the other hand, most of the actual economic series that the models try to replicate display significant variability and high persistence (as measured by serial correlation). As summarized by King and Rebelo (1999, pp. 34-37) a common result in the literature is that the standard models have only a partial success in reproducing the actual behavior of the economy, unless they assume ad hoc frictions. In particular the standard modeling methodology has often faced two limits. First, many of these models have a weak internal propagation mechanism. The effects of short-lived exogenous shocks do not propagate much over time, contrary to what we see in the data. In other words, the standard models require highly persistent shocks (i.e. shock with high serial correlation). Second, most of the models do not have strong amplification mechanisms. This means that often they cannot obtain large endogenous fluctuations from small exogenous shocks.

b. The Role of Evolutionary Dynamics

We think adaptive/evolutionary dynamics can improve the performance of business cycle models. First, an adaptive model could provide a stronger, more realistic propagation mechanism. The intuition is that it takes many periods for the population to adjust to shocks
and thus the effects of a change in steady state will propagate over time. Second, we expect that some evolutionary dynamics can provide also a stronger amplification mechanisms. For instance Rock-Paper-Scissor type dynamics allow for minor changes to the steady state to cause large drifts in the population state.

We explore these ideas simulating dynamics in games with random shocks to the payoff structure. The example we choose is the general $2 \times 2$ game (with two populations). The game is defined by four payoff matrices. We denote the steady state value of these matrices by: $\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}$. Since we study discrete time replicator dynamics the matrices are appropriately scaled. Then we introduce random shocks in the following way: each element of each payoff matrix is affected by a shock in each period. Shocks affect the matrixes in the following way:

$$X_t = \begin{bmatrix} \pi_{11} \times e^{\tau_1(X_{11})} & \pi_{12} \times e^{\tau_1(X_{12})} \\ \pi_{21} \times e^{\tau_1(X_{21})} & \pi_{22} \times e^{\tau_1(X_{22})} \end{bmatrix}$$

where $\pi_{ij}$ is the respective element of matrix $X$ and:

$$e_t(X_{ij}) \sim iid N(0; \sigma^2)$$

Note that we assume no autocorrelation in the shocks. Moreover, shocks hit the model in every period (as it is usual in the standard business cycle models).

Let $s$ be the state vector in the first population and $r$ the state vector for the second one. For strategy $i$ in the first population the payoff is:

$$w_{it} = e_i \times A_t \times s_t + e_i \times B_t \times r_t$$

where $e_1 = \begin{bmatrix} 1 & 0 \end{bmatrix}$ and $e_2 = \begin{bmatrix} 0 & 1 \end{bmatrix}$. Similarly for strategy $i$ in the second population:

$$u_{it} = e_i \times C_t \times s_i + e_i \times D_t \times r_i$$

We study the behavior of stable interior steady states, since corner solutions are affected only by very large shocks to the payoffs. We contrast two types of evolutionary game: one with an interior sink and one with an interior stable center. In both cases we use the stochastic NE as a benchmark for the evolutionary behavior. The basic idea is to compare
the performance of a model based on static optimal choice (the NE series) and that of an adaptive model (the evolutionary state series).

**Simulations with an Interior Sink**

We use arbitrary $\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}$ with the only constraint that in the steady state there be an interior sink. We set $\sigma^2 = 0.001$. Then we run the simulation for 600 periods. We focus on one strategy share in one population (in order to make the graph simpler). We compare the series of the strategy share in the (stochastic) NE with the series of the strategy share obtained from replicator dynamics. We want the system to be close to the steady state because we the impact of the arbitrary initial condition is not of interest. Thus we plot only the last 100 periods.

![Graph](image)

**Figure 8: Simulated Series - Sink**

As expected the evolutionary series displays much more persistence than the NE series. The reason is that it takes time to agent to adjust their choices. However, the evolutionary series has almost no variation compared to the NE.
Simulations with an Interior Stable Center

A more interesting situation is a stable center. Again we simulate the model for 600 periods and reproduce the two series only for the last 100.

![Graph showing evolutionary dynamics and Nash equilibrium](image)

Figure 9: Simulated Series - Center

It is easy to see that the evolutionary series has more persistence than the NE series and as much variability as the NE, if not more. It seems that evolutionary dynamics with a stable center, such as a rock-paper-scissors game, are suited particularly well for modeling business cycles.

c. Problems in Modeling Cycles with Evolutionary Games

However using a RPS set up to model business cycles involves a number of complications. First, one needs to set up an economic model with three or more strategies and specify the payoff functions to be consistent with economic theory. These payoff functions typically involve non-linear relations (such as supply and demand functions). Then, one needs to find the steady state(s) of such a model under replicator dynamics. The next step would be to linearize the model around the steady state(s) and check the conditions for a stable center. A stable center arises when the payoff structure satisfies certain intransitivity relations. These
conditions have to ensure not only that the linearized system has eigenvalues with imaginary parts but also that their real part is negative. These conditions impose very stringent restrictions on the model. It is difficult to justify all of these restrictions using standard assumptions from economic theory. Because of these difficulties we decided to use a different strategy to model cycles. As discussed above, the key element of this alternative strategy is the use of belief learning with short memory.
BIBLIOGRAPHY


