Tutorial Section Times

• All sections meet in lab or outside at the white boards in Jack’s Lounge

• Monday 10:45-11:45 am (Patrick Ellis)
• Monday 7-8 pm (Ali Adabi)
• Tuesday 12:45-1:45 pm (Patrick Ellis)
• Thursday 3-4 pm (Ali Adabi)
Why Mesh Analysis

• More convenient (often) and results in less equations to solve

• A mesh is a loop that contains no loops within it

• Mesh analysis applies only to planar circuits -- the circuit diagram can be written on a flat piece of paper without jumping over wires.
Mesh Current Analysis

Figure 2.31 Circuit for illustrating the mesh-current method of circuit analysis.
Choosing the Mesh Currents

When several mesh currents flow through one element, we consider the current in that element to be the algebraic sum of the mesh currents.

Sometimes it is said that the mesh currents are defined by “soaping the window panes.”
Multiple mesh currents flow through $R_2$ in (a) and (b) plus other elements.
Writing Equations to Solve for Mesh Currents

If a network contains only resistances and independent voltage sources, we can write the required equations by following each current around its mesh and applying KVL.
Mesh Current Steps

1. Assign mesh currents to $i_j$ to all the meshes
2. Apply KVL, using Ohm’s law to express $v$ using the mesh currents ($v = i R$)
3. Solve the resulting equations in standard form using Cramer’s Rule or other method if the equations are simple
Using this pattern for mesh 1 of

\[ R_2 (i_1 - i_3) + R_3 (i_1 - i_2) - v_A = 0 \]

For mesh 2, we obtain

\[ R_3 (i_2 - i_1) + R_4 i_2 + v_B = 0 \]

For mesh 3, we have

\[ R_2 (i_3 - i_1) + R_1 i_3 - v_B = 0 \]
For the circuit below we have 4 meshes

\[ R_1 i_1 + R_2 (i_1 - i_4) + R_4 (i_1 - i_2) - v_A = 0 \]

\[ R_5 i_2 + R_4 (i_2 - i_1) + R_6 (i_2 - i_3) = 0 \]

\[ R_7 i_3 + R_6 (i_3 - i_2) + R_8 (i_3 - i_4) = 0 \]

\[ R_3 i_4 + R_2 (i_4 - i_1) + R_8 (i_4 - i_3) = 0 \]
Solve for the mesh currents:

\[ 20i_1 + 10(i_1 - i_2) - 150 = 0 \]
\[ 15i_2 + 100 + 10(i_2 - i_1) = 0 \]
\begin{align*}
20i_1 + 10(i_1 - i_2) - 150 &= 0 \\
15i_2 + 100 + 10(i_2 - i_1) &= 0
\end{align*}

Putting the equations into the standard format:

\begin{align*}
30i_1 - 10i_2 &= 150 \\
-10i_1 + 25i_2 &= -100
\end{align*}
Use Cramer’s Rule

\[ 30i_1 - 10i_2 = 150 \]
\[ -10i_1 + 25i_2 = -100 \]

\[ i_1 = \frac{\begin{vmatrix} 150 & -10 \\ -100 & 25 \\ 30 & -10 \\ -10 & 25 \end{vmatrix}}{\begin{vmatrix} 30 & -10 \\ 30 & 25 \\ -10 & 25 \end{vmatrix}} = \frac{(150)(25) - (-100)(-10)}{(30)(25) - (-10)(-10)} = \frac{2750}{650} = 4.23 \]

\[ i_2 = \frac{\begin{vmatrix} 30 & 150 \\ -10 & -100 \\ 30 & -10 \\ -10 & 25 \end{vmatrix}}{\begin{vmatrix} 30 & -10 \\ 30 & 25 \\ -10 & 25 \end{vmatrix}} = \frac{(30)(-100) - (-10)(150)}{(30)(25) - (-10)(-10)} = \frac{-1500}{650} = -2.31 \]
Find the current through the 10Ω resistor using mesh currents:

\[ 5i_1 + 10(i_1 - i_2) - 100 = 0 \]
\[ 10(i_2 - i_1) + 7i_2 + 3i_2 = 0 \]
\[ 15i_1 - 10i_2 = 100 \]
\[ -10i_1 + 20i_2 = 0 \]
\begin{align*}
15i_1 - 10i_2 &= 100 \\
-10i_1 + 20i_2 &= 0 \\
\end{align*}

\[ i_1 = \frac{100 \begin{vmatrix} 100 & -10 \\ 0 & 20 \\ 15 & -10 \\ -10 & 20 \end{vmatrix} = \frac{(100)(20) - (0)(-10)}{(15)(20) - (-10)(-10)} = \frac{2000}{200} = 10 }{100 \begin{vmatrix} 100 & -10 \\ 0 & 20 \\ 15 & -10 \\ -10 & 20 \end{vmatrix} = \frac{(100)(20) - (0)(-10)}{(15)(20) - (-10)(-10)} = \frac{2000}{200} = 10 \]

\[ i_2 = \frac{15 \begin{vmatrix} 15 & 100 \\ -10 & 0 \\ 15 & -10 \\ -10 & 20 \end{vmatrix} = \frac{(15)(0) - (-10)(100)}{(15)(20) - (-10)(-10)} = \frac{1000}{200} = 5 }{15 \begin{vmatrix} 15 & 100 \\ -10 & 0 \\ 15 & -10 \\ -10 & 20 \end{vmatrix} = \frac{(15)(0) - (-10)(100)}{(15)(20) - (-10)(-10)} = \frac{1000}{200} = 5 \]

\[ i_{10\Omega} = i_1 - i_2 = 10 - 5 = 5 \text{A} \]

Figure 2.34 Circuit of Exercise 2.16.
Find the current through the 10Ω resistor using node currents.

\[
\nu_1 = 100V
\]
\[
\frac{100 - \nu_2}{5} = \frac{\nu_2}{10} + \frac{\nu_2}{10}
\]
\[
\frac{\nu_2 - 100}{5} + \frac{\nu_2}{10} + \frac{\nu_2}{10} = 0
\]
\[
\frac{2\nu_2 - 200 + 2\nu_2}{10} = 0
\]
\[
4\nu_2 = 200
\]
\[
\nu_2 = 50
\]
\[
i_3 = \frac{\nu_2}{10} = 5A
\]
Find the current through the 10Ω resistor by combining resistances in series and parallel:

\[ i_1 = \frac{100V}{10\Omega} = 10A, \]

*equally split between two 10Ω resistors in parallel* \( \rightarrow i_3 = 5A \)
Mesh Currents in Circuits Containing Current Sources

A common mistake is to assume that the voltages across current sources are zero. (We don’t know before we analyze.)

Figure 2.35 In this circuit, we have $i_1 = 2 \text{ A}$. 
\( i_1 = 2A \)

**KVL for loop 2:** \( 10(i_2 - i_1) + 5i_2 + 10 = 0 \)

\[
10i_2 - 20 + 5i_2 + 10 = 0
\]

\[
15i_2 = 10
\]

\[
i_2 = \frac{10}{15} = \frac{2}{3} A
\]
Figure 2.36 A circuit with a current source common to two meshes.
When we have a current source in common, we combine meshes 1 and 2 into a **supermesh**. In other words, we write a KVL equation around the periphery of meshes 1 and 2 combined.

\[ 1i_1 + 2(i_1 - i_3) + 4(i_2 - i_3) + 10 = 0 \]

**Mesh 3:**

\[ 3i_3 + 4(i_3 - i_2) + 2(i_3 - i_1) = 0 \]

\[ i_2 - i_1 = 5 \quad \text{Additional Eq. from supermesh} \]
Solve for the mesh currents (current and voltage sources):

\[ i_1 = -5 \, A \]
\[ 10(i_2 - i_1) + 5i_2 - 100 = 0 \]
\[ 15i_2 = 50 \]
\[ i_2 = \frac{50}{15} \]

**Figure 2.37** The circuit for Exercise 2.18.
To solve for the mesh currents we use a supermesh around the perimeter of the circuit:

\[-20 + 4i_1 + 6i_2 + 2i_2 = 0 \rightarrow 4i_1 + 8i_2 = 20\]

\[\frac{v_x}{4} = i_2 - i_1\]  \hspace{1cm} \text{Constraint Eq.}

\[v_x = 2i_2\]

**Substituting:**

\[\frac{i_2}{2} = i_2 - i_1 \rightarrow i_1 - \frac{i_2}{2} = 0\]
\[ 4i_1 + 8i_2 = 20 \]
\[ i_1 - 0.5i_2 = 0 \]

\[
\begin{align*}
i_1 &= \begin{vmatrix} 20 & 8 \\ 0 & -0.5 \\ 4 & 8 \\ 1 & -0.5 \end{vmatrix} = \frac{(20)(-0.5) - (0)(8)}{(4)(-0.5) - (1)(8)} = \frac{-10}{-10} = 1 \\
i_2 &= \begin{vmatrix} 4 & 20 \\ 1 & 0 \\ 4 & 8 \\ 1 & -0.5 \end{vmatrix} = \frac{(4)(0) - (1)(20)}{(4)(-0.5) - (1)(8)} = \frac{-20}{-10} = 2
\end{align*}
\]
Solve for the mesh currents:

\[ 10(i_1 - i_2) - 25 + 10(i_1 - i_3) = 0 \quad \rightarrow \quad 20i_1 - 10i_2 - 10i_3 = 25 \]
\[ 10(i_2 - i_1) + 20(i_2 - i_3) + 20i_2 = 0 \quad \rightarrow \quad -10i_1 + 50i_2 - 20i_3 = 0 \]
\[ 5i_3 + 20(i_3 - i_2) + 10(i_3 - i_1) = 0 \quad \rightarrow \quad -10i_1 - 20i_2 + 35i_3 = 0 \]

The equations are now in standard form and can be solved using Cramer’s method.
\[20i_1 - 10i_2 - 10i_3 = 25\]
\[-10i_1 + 50i_2 - 20i_3 = 0\]
\[-10i_1 - 20i_2 + 35i_3 = 0\]

\[
i_1 = \begin{vmatrix}
25 & -10 & -10 \\
0 & 50 & -20 \\
0 & -20 & 35 \\
\end{vmatrix} = \frac{(25)(50)(35) - (25)(-20)(-20)}{(20)(50)(35) + (-10)(-20)(-10) + (-10)(-10)(-20) - [(\cdot)(50)(\cdot) + (\cdot)(\cdot)(\cdot) + (\cdot)(\cdot)(\cdot)]} = \frac{33750}{14500} = 2.33
\]

\[
i_2 = \begin{vmatrix}
20 & 25 & -10 \\
-10 & 0 & -20 \\
-10 & 0 & 35 \\
\end{vmatrix} = \frac{(25)(-20)(-10) - (25)(-10)(35)}{(20)(50)(35) + (-10)(-20)(-10) + (-10)(-10)(-20) - [(\cdot)(50)(\cdot) + (\cdot)(\cdot)(\cdot) + (\cdot)(\cdot)(\cdot)]} = \frac{13750}{14500} = 0.948
\]

\[
i_3 = \begin{vmatrix}
20 & -10 & 25 \\
-10 & 50 & 0 \\
-10 & -20 & 0 \\
\end{vmatrix} = \frac{(25)(-20)(-10) - (25)(50)(-10)}{(20)(50)(35) + (-10)(-20)(-10) + (-10)(-10)(-20) - [(\cdot)(50)(\cdot) + (\cdot)(\cdot)(\cdot) + (\cdot)(\cdot)(\cdot)]} = \frac{17500}{14500} = 1.21
\]
Mesh-Current Analysis

1. If necessary, redraw the network without crossing conductors or elements. Then define the mesh currents flowing around each of the open areas defined by the network. For consistency, we usually select a clockwise direction for each of the mesh currents, but this is not a requirement.
2. Write network equations, stopping after the number of equations is equal to the number of mesh currents. First, use KVL to write voltage equations for meshes that do not contain current sources. Next, if any current sources are present, write expressions for their currents in terms of the mesh currents. Finally, if a current source is common to two meshes, write a KVL equation for the supermesh.
3. If the circuit contains dependent sources, find expressions for the controlling variables in terms of the mesh currents. Substitute into the network equations, and obtain equations having only the mesh currents as unknowns.
4. Put the equations into standard form. Solve for the mesh currents by use of determinants or other means.

5. Use the values found for the mesh currents to calculate any other currents or voltages of interest.