Section 13.2 Mutual Inductance

13.1 For the three coupled coils in Fig. 13.72, calculate the total inductance.

![Figure 13.72](image)

For Prob. 13.1.

13.2 Using Fig. 13.73, design a problem to help other students better understand mutual inductance.

![Figure 13.73](image)

For Prob. 13.2.

13.3 Two coils connected in series-aiding fashion have a total inductance of 250 mH. When connected in a series-opposing configuration, the coils have a total inductance of 150 mH. If the inductance of one coil \( L_1 \) is three times the other, find \( L_1, L_2, \) and \( M \). What is the coupling coefficient?

13.4 (a) For the coupled coils in Fig. 13.74(a), show that

\[
L_{eq} = L_1 + L_2 + 2M
\]

(b) For the coupled coils in Fig. 13.74(b), show that

\[
L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}
\]

![Figure 13.74](image)

For Prob. 13.4.

13.5 Two coils are mutually coupled, with \( L_1 = 25 \) mH, \( L_2 = 60 \) mH, and \( k = 0.5 \). Calculate the maximum possible equivalent inductance if:
(a) the two coils are connected in series
(b) the coils are connected in parallel

13.6 The coils in Fig. 13.75 have \( L_1 = 40 \) mH, \( L_2 = 5 \) mH, and coupling coefficient \( k = 0.6 \). Find \( i_1(t) \) and \( v_2(t) \), given that \( v_1(t) = 10 \cos \omega t \) and \( i_2(t) = 2 \sin \omega t \), \( \omega = 2000 \) rad/s.

![Figure 13.75](image)

For Prob. 13.6.

13.7 For the circuit in Fig. 13.76, find \( V_o \).

![Figure 13.76](image)

For Prob. 13.7.

13.8 Find \( v(t) \) for the circuit in Fig. 13.77.

![Figure 13.77](image)

For Prob. 13.8.

13.9 Find \( V_x \) in the network shown in Fig. 13.78.

![Figure 13.78](image)

For Prob. 13.9.

1 Remember, unless otherwise specified, assume all values of currents and voltages are rms.
13.10 Find \( v_o \) in the circuit of Fig. 13.79.

\[ 120 \cos 2t \quad \text{V} \quad 2 \text{H} \quad 2 \text{H} \quad 0.5 \text{F} \quad v_o \]

**Figure 13.79**
For Prob. 13.10.

13.11 Use mesh analysis to find \( i_x \) in Fig. 13.80, where

\[ i_x = 6 \cos(600t) \quad \text{A} \quad \text{and} \quad v_x = 165 \cos(600t + 30^\circ) \]

\[ \begin{array}{ccc}
     & 800 \text{mH} & 12 \mu \text{F} \\
& \downarrow & 150 \Omega \\
& \downarrow & 200 \Omega \\
& 600 \text{mH} & \end{array} \]

**Figure 13.80**
For Prob. 13.11.

13.12 Determine the equivalent \( L_{eq} \) in the circuit of Fig. 13.81.

\[ \begin{array}{ccc}
     & 2 \text{H} & 4 \text{H} \\
& \downarrow & \downarrow \\
& 6 \text{H} & 8 \text{H} & 10 \text{H} \\
& \downarrow & \downarrow & \downarrow \\
& 120 \Omega \quad \text{V} \quad j2 \Omega \quad j2 \Omega & \\
& \downarrow & \downarrow & \downarrow \\
& j4 \Omega & j6 \Omega & 2 \Omega & 60 \Omega & Z_L & 12 \text{mH} & 30 \text{mH} & \end{array} \]

**Figure 13.81**
For Prob. 13.12.

13.13 For the circuit in Fig. 13.82, determine the impedance seen by the source.

**Figure 13.82**
For Prob. 13.13.

13.14 Obtain the Thevenin equivalent circuit for the circuit in Fig. 13.83 at terminals \( a-b \).

**Figure 13.83**
For Prob. 13.14.

13.15 Find the Norton equivalent for the circuit in Fig. 13.84 at terminals \( a-b \).

**Figure 13.84**
For Prob. 13.15.

13.16 Obtain the Norton equivalent at terminals \( a-b \) of the circuit in Fig. 13.85.

**Figure 13.85**
For Prob. 13.16.

13.17 In the circuit of Fig. 13.86, \( Z_L \) is a 15-mH inductor having an impedance of \( j40 \) \( \Omega \). Determine \( Z_{in} \) when \( k = 0.6 \).

**Figure 13.86**
For Prob. 13.17.
3. Consider the circuit below:

\[ 40 \cos 2t \text{ V} \]

Determine the coupling coefficient \( k \) & the total energy stored in the coupled inductors as determined by the magnitudes of the currents \( I_1 \) & \( I_2 \).
4. Energy in a coupled circuit

Consider the circuit below.

\[ v_1 \rightarrow \rightarrow M \rightarrow i_2 \leftarrow \rightarrow v_2 \]

\[ j10 \Omega \]

\[ j5 \Omega \]

\[ v_1 \]

\[ v_2 \]

Coupling coefficient \[ k = 0.5 \]

Find the instantaneous energy stored in the circuit if

\[ i_1 = 10 \cos(\omega t) \quad \& \quad i_2 = 5 \cos(\omega t) \]

What would the energy be if the dot on the right-hand winding were moved to the bottom of the winding?
13.25 For the network in Fig. 13.94, find $Z_{ab}$ and $I_o$.

For Prob. 13.25.

![Figure 13.94](image)

13.26 Find $I_o$ in the circuit of Fig. 13.95. Switch the dot on the winding on the right and calculate $I_o$ again.


![Figure 13.95](image)

13.27 Find the average power delivered to the 50-Ω resistor in the circuit of Fig. 13.96.

For Prob. 13.27.

![Figure 13.96](image)

*13.28 In the circuit of Fig. 13.97, find the value of $X$ that will give maximum power transfer to the 20-Ω load.

For Prob. 13.28.

![Figure 13.97](image)

Section 13.4 Linear Transformers

13.29 In the circuit of Fig. 13.98, find the value of the coupling coefficient $k$ that will make the 10-Ω resistor dissipate 320 W. For this value of $k$, find the energy stored in the coupled coils at $t = 1.5$ s.

For Prob. 13.29.

![Figure 13.98](image)

13.30 (a) Find the input impedance of the circuit in Fig. 13.99 using the concept of reflected impedance.

(b) Obtain the input impedance by replacing the linear transformer by its $T$ equivalent.

For Prob. 13.30.

![Figure 13.99](image)

13.31 Using Fig. 13.100, design a problem to help other students better understand linear transformers and how to find $T$-equivalent and $Π$-equivalent circuits.

For Prob. 13.31.

![Figure 13.100](image)

*13.32 Two linear transformers are cascaded as shown in Fig. 13.101. Show that

$$Z_{in} = \frac{\omega^2 R L_a^2 + L_a L_b - M_a^2}{\omega^2 (L_a L_b + L_b^2 - M_b^2) - j \omega R L_a + L_b}$$

For Prob. 13.32.
Section 13.5  Ideal Transformers

13.36 As done in Fig. 13.32, obtain the relationships between terminal voltages and currents for each of the ideal transformers in Fig. 13.105.

Figure 13.105
For Prob. 13.36.

13.37 A 480/2,400-V rms step-up ideal transformer delivers 50 kW to a resistive load. Calculate:
(a) the turns ratio
(b) the primary current
(c) the secondary current

13.38 Design a problem to help other students better understand ideal transformers.

13.39 A 1,200/240-V rms transformer has impedance $60^\circ/30^\circ \Omega$ on the high-voltage side. If the transformer is connected to a $0.8/100\Omega$ load on the low-voltage side, determine the primary and secondary currents when the transformer is connected to 1200 V rms.
13.40 The primary of an ideal transformer with a turns ratio of 5 is connected to a voltage source with Thevenin parameters $v_{th} = 10 \cos 2000t$ V and $R_{th} = 100$ Ω. Determine the average power delivered to a 200-Ω load connected across the secondary winding.

13.41 Determine $I_1$ and $I_2$ in the circuit of Fig. 13.106.

![Figure 13.106](image)

**Figure 13.106**
For Prob. 13.41.

13.42 For the circuit in Fig. 13.107, determine the power absorbed by the 2-Ω resistor. Assume the 80 V is an rms value.

![Figure 13.107](image)

**Figure 13.107**
For Prob. 13.42.

13.43 Obtain $V_1$ and $V_2$ in the ideal transformer circuit of Fig. 13.108.

![Figure 13.108](image)

**Figure 13.108**
For Prob. 13.43.

13.44 In the ideal transformer circuit of Fig. 13.109, find $i_1(t)$ and $i_2(t)$.

![Figure 13.109](image)

**Figure 13.109**
For Prob. 13.44.

13.45 For the circuit shown in Fig. 13.110, find the value of the average power absorbed by the 8-Ω resistor.

![Figure 13.110](image)

**Figure 13.110**
For Prob. 13.45.

13.46 (a) Find $I_1$ and $I_2$ in the circuit of Fig. 13.111 below.

(b) Switch the dot on one of the windings. Find $I_1$ and $I_2$ again.

13.47 Find $v(t)$ for the circuit in Fig. 13.112.

![Figure 13.111](image)

**Figure 13.111**
For Prob. 13.46.
A transformer is used to match an amplifier with an 8-Ω load as shown in Fig. 13.119. The Thevenin equivalent of the amplifier is: $V_{TH} = 10$ V, $Z_{TH} = 128$ Ω.

(a) Find the required turns ratio for maximum energy transfer.
(b) Determine the primary and secondary currents.
(c) Calculate the primary and secondary voltages.

Figure 13.119
For Prob. 13.54.

For the circuit in Fig. 13.120, calculate the equivalent resistance.

Figure 13.120
For Prob. 13.55.

Find the power absorbed by the 10-Ω resistor in the ideal transformer circuit of Fig. 13.121.

Figure 13.121
For Prob. 13.56.

For the ideal transformer circuit of Fig. 13.122 below, find:

(a) $I_1$ and $I_2$.
(b) $V_1$, $V_2$, and $V_o$.
(c) the complex power supplied by the source.

Figure 13.122
For Prob. 13.57.

Determine the average power absorbed by each resistor in the circuit of Fig. 13.123.

Figure 13.123
For Prob. 13.58.

In the circuit of Fig. 13.124, let $v_s = 160 \cos 1000t$.
Find the average power delivered to each resistor.

Figure 13.124
For Prob. 13.59.

Refer to the circuit in Fig. 13.125 on the following page.

(a) Find currents $I_1$, $I_2$, and $I_3$.
(b) Find the power dissipated in the 40-Ω resistor.
Section 13.6  Ideal Autotransformers

13.66 Design a problem to help other students better understand how the ideal autotransformer works.

13.67 An autotransformer with a 40 percent tap is supplied by a 400-V, 60-Hz source and is used for step-down operation. A 5-kVA load operating at unity power factor is connected to the secondary terminals. Find:
(a) the secondary voltage
(b) the secondary current
(c) the primary current

13.68 In the ideal autotransformer of Fig. 13.131, calculate $I_1$, $I_2$, and $I_o$. Find the average power delivered to the load.

13.70 In the ideal transformer circuit shown in Fig. 13.133, determine the average power delivered to the load.

13.71 In the autotransformer circuit in Fig. 13.134, show that
\[ Z_{in} = \left( 1 + \frac{N_1}{N_2} \right)^2 Z_L \]

*13.69 In the circuit of Fig. 13.132, $Z_L$ is adjusted until maximum average power is delivered to $Z_L$. Find $Z_L$ and the maximum average power transferred to it. Take $N_1 = 600$ turns and $N_2 = 200$ turns.

13.72 In order to meet an emergency, three single-phase transformers with 12,470/7,200 V rms are connected in Δ-Y to form a three-phase transformer which is fed by a 12,470-V transmission line. If the transformer supplies 60 MVA to a load, find:
(a) the turns ratio for each transformer,
(b) the currents in the primary and secondary windings of the transformer,
(c) the incoming and outgoing transmission line currents.

13.73 Figure 13.135 on the following page shows a three-phase transformer that supplies a Y-connected load.
(a) Identify the transformer connection.
(b) Calculate currents $I_L$ and $I_o$.
(c) Find the average power absorbed by the load.
13.84 Determine $I_1$, $I_2$, and $I_3$ in the ideal transformer circuit of Fig. 13.145 using PSpice.

![Figure 13.145](image)

For Prob. 13.84.

**Section 13.9 Applications**

13.85 A stereo amplifier circuit with an output impedance of 7.2 kΩ is to be matched to a speaker with an input impedance of 8 Ω by a transformer whose primary side has 3,000 turns. Calculate the number of turns required on the secondary side.

13.86 A transformer having 2,400 turns on the primary and 48 turns on the secondary is used as an impedance-matching device. What is the reflected value of a 3-Ω load connected to the secondary?

13.87 A radio receiver has an input resistance of 300Ω. When it is connected directly to an antenna system with a characteristic impedance of 75 Ω, an impedance mismatch occurs. By inserting an impedance-matching transformer ahead of the receiver, maximum power can be realized. Calculate the required turns ratio.

13.88 A step-down power transformer with a turns ratio of $n = 0.1$ supplies 12.6 V rms to a resistive load. If the primary current is 2.5 A rms, how much power is delivered to the load?

13.89 A 240/120-V rms power transformer is rated at 10 kVA. Determine the turns ratio, the primary current, and the secondary current.

13.90 A 4-kVA, 2,400/240-V rms transformer has 250 turns on the primary side. Calculate:
   (a) the turns ratio,
   (b) the number of turns on the secondary side,
   (c) the primary and secondary currents.

13.91 A 25,000/240-V rms distribution transformer has a primary current rating of 75 A.
   (a) Find the transformer kVA rating.
   (b) Calculate the secondary current.

13.92 A 4,800-V rms transmission line feeds a distribution transformer with 1,200 turns on the primary and 28 turns on the secondary. When a 10-Ω load is connected across the secondary, find:
   (a) the secondary voltage,
   (b) the primary and secondary currents,
   (c) the power supplied to the load.

**Comprehensive Problems**

13.93 A four-winding transformer (Fig. 13.146) is often used in equipment (e.g., PCs, VCRs) that may be operated from either 110 V or 220 V. This makes the equipment suitable for both domestic and foreign use. Show which connections are necessary to provide:
   (a) an output of 14 V with an input of 110 V,
   (b) an output of 50 V with an input of 220 V.

![Figure 13.146](image)

For Prob. 13.93.

13.94 A 440/110-V ideal transformer can be connected to become a 550/440-V ideal autotransformer. There are four possible connections, two of which are wrong. Find the output voltage of:
   (a) a wrong connection,
   (b) the right connection.

13.95 Ten bulbs in parallel are supplied by a 7,200/120-V transformer as shown in Fig. 13.147, where the bulbs are modeled by the 144-Ω resistors. Find:
   (a) the turns ratio $n$,
   (b) the current through the primary winding.

![Figure 13.147](image)

For Prob. 13.95.