Operational Amplifier II

- The op amp is capable of many math operations, such as addition, subtraction, multiplication, differentiation, and integration

- There are five terminals found on all op-amps
  - The inverting input
  - The noninverting input
  - The output
  - The positive and negative power supplies
Voltage Saturation

- As an ideal source, the output voltage would be unlimited.
- In reality, one cannot expect the output to exceed the supply voltages.
- When an output should exceed the possible voltage range, the output remains at either the maximum or minimum supply voltage.
- This is called saturation.
- Outputs between these limiting voltages are referred to as the linear region.
LM741 Op Amp

There are 20 BJTs, 11 Rs and 1 C.
Inverting Amplifier

- The first useful op-amp circuit that we will consider is the inverting amplifier.
- Here the noninverting input is grounded.
- The inverting terminal is connected to the output via a feedback resistor, $R_f$.
- The input is also connected to the inverting terminal via another resistor, $R_1$.
Non-Inverting Amplifier

- Another important op-amp circuit is the noninverting amplifier
- The basic configuration of the amplifier is the same as the inverting amplifier
- Except that the input and the ground are switched
- Once again applying KCL to the inverting terminal gives:

\[ i_1 = i_2 \Rightarrow \frac{0 - v_1}{R_1} = \frac{v_1 - v_o}{R_f} \]
Summing amplifiers (mixers)

Summing Amplifier is an op amp circuit that combines several inputs and produces an output that is the weighted sum of the inputs.

\[ v_o = -\left( \frac{R_f}{R_1} v_1 + \frac{R_f}{R_2} v_2 + \frac{R_f}{R_3} v_3 \right) \]

\[ i = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} \]

\[ R_g = \sum_{i=1}^{3} \frac{R_i}{R_i} V_i \]
Summing Amplifier Example

Calculate $v_o$ and $i_o$ in the op amp mixer circuit shown below.

KCL at node a dictates that

$$\frac{2}{5K} + \frac{1}{2.5K} = -\frac{v_o}{10K}$$

$4 + 4 = -v_o$, thus $v_o = -8$ V

$$i_o = 8 \left( \frac{1}{10K} + \frac{1}{2K} \right) = -4.8mA$$

\[ \begin{align*}
  v_o &= - \left( \frac{2}{5K} + \frac{1}{2.5K} \right) 10K \\
  \frac{v_o}{2K} &= \frac{v_o}{2K} + \left( -\frac{2}{10K} - \frac{1}{2.5K} \right)
\end{align*} \]
Differential Amplifier with Op Amp

- Difference amplifier is a device that amplifies the difference between two inputs but rejects any signals common to the two inputs (common mode interference).

Analysis: Apply KCL at both nodes a & b and solve for $v_o$.

$$v_o = \frac{R_2}{R_1(1 + R_3/R_4)} v_2 - \frac{R_2}{R_1} v_1 \Rightarrow v_o = v_2 - v_1 \text{, if } \frac{R_2}{R_1} = \frac{R_3}{R_4} = 1$$

Reject common mode
DAC Application

- Digital-to Analog Converter (DAC): it is a device which transforms digital signals into analog form.

Four-bit DCA: (a) block diagram

\[ V_o = \frac{R_f}{R_1} V_1 + \frac{R_f}{R_2} V_2 + \frac{R_f}{R_3} V_3 + \frac{R_f}{R_4} V_4 \]

where

- \( V_1 \) – MSB, \( V_4 \) – LSB
- \( V_1 \) to \( V_4 \) are either 0 or 1 V
- \( \nu_o = \sum_{k=0}^{N} C_k \)
- here \((\nu_1, ..., \nu_N) = (b_1, ..., b_N)\),
- \( C_k = \frac{R_f}{R_k} \) and \( R_k = R_1 2^{k-1} \)
DAC Circuit Example

For the circuit shown below, calculate $v_o$. If $v_1 = 0\,\text{V}$, $v_2 = 1\,\text{V}$ and $v_3 = 1\,\text{V}$.

Binary sequence of (0 1 1)

For $(1\ 1\ 1)$, $v_o = -\left(\frac{1}{10}10 + \frac{1}{20}10 + \frac{1}{40}10\right) = -1.75\,\text{V}$.  

Ans.: $-0.75\,\text{V}$
Problem 5.84

A 4-bit R-2R ladder DAC is shown below. Show that the output voltage can be expressed as

\[-V_0 = R_f \left( \frac{V_1}{2R} + \frac{V_2}{4R} + \frac{V_3}{8R} + \frac{V_4}{16R} \right)\]

KCL at the inverting input node:
(virtual ground)

Apply the superposition principle-
1) Output voltage due to $V_1$ only
$V_o$ (v1 only) = $\frac{V_1}{2R} \cdot R_f$
2) Output voltage due to $V_2$ only
$V_o$ (v2 only) =
At node X, \((V2-Vx)/2R = Vx/2R + Vx/R\)
$Vx = V2/4$, \(V0 = -Rf \cdot (V2/4)/R = -Rf/4R\), repeat for
$V3$, $V4$ and add them up.

(Virtual ground)
Source-Free RC Circuit Example

Example 2

The switch in circuit below is opened at \( t = 0 \), find \( v(t) \) for \( t \geq 0 \).

\[
\ln \frac{v}{8} + 2t = 0
\]

\[
\ln \frac{v}{8} = -2t
\]

\[
e^{\ln \frac{v}{8}} = e^{-2t}
\]

\[
v = 8e^{-2t}
\]

Answer: \( V(t) = 8e^{-2t} \, V \)

Please refer to lecture or textbook for more detail elaboration.

\[
V(0^-) = 8 \, V
\]

\[
V(0^+) = 8 \, V
\]
Source-Free RC Circuit Example

Example 1
Refer to the circuit below, determine $v_C$, $v_x$, and $i_o$ for $t \geq 0$. Assume that $v_C(0) = 30$ V.

Answer: $v_C = 30e^{-0.25t}$ V; $v_x = 10e^{-0.25t}$; $i_o = -2.5e^{-0.25t}$ A
How to solve \( \frac{dv}{dt} = -\frac{v-V_s}{RC} \)?

We first note that \( dv = d(v-V_s) \) and rewrite the equation above:

\[
\frac{d(v-V_s)}{dt} = -\frac{v-V_s}{RC}
\]

Integrating both sides, we get

\[
\ln(v-V_s) \big|_{(v_0-V_s)} \text{ to } (v-V_s) = -\frac{1}{RC} (t-t_0)
\]

\[
V-V_s = (v_0-V_s) e^{-\frac{t-t_0}{RC}}
\]

Finally, \( v(t) = V_s + (v_0-V_s) e^{-\frac{t-t_0}{RC}} \)

\( V(t_0) = v_0, \text{ and } v(\infty) = V_s \)
Step-Response of a RC Circuit

- Integrating both sides and considering the initial conditions, the solution of the equation is:

\[
V(t) = \begin{cases} 
V_0 & t < 0 \\
V_s + (V_0 - V_s)e^{-t/\tau} & t > 0 
\end{cases}
\]

- Final value at \( t \to \infty \)
- Initial value at \( t = 0 \)
- Source-free Response

Complete Response = Natural response (stored energy) + Forced Response (independent source)

\[
= V_0 e^{-t/\tau} + V_s(1 - e^{-t/\tau})
\]
Step-Response of a RL Circuit

The switch in the circuit shown below has been closed for a long time. It opens at \( t = 0 \).
Find \( i(t) \) for \( t > 0 \).

At \( t = 0 \), \( i = 2 \) A according to the current division law.
KCL at the top right node is

\[
3 = 1 + \frac{5i + 1.5 \frac{di}{dt}}{10}
\]

\[
30 = 15i + 1.5 \frac{di}{dt} \quad \text{or} \quad \frac{di}{dt} + 10i = 20 \quad \text{or} \quad \frac{di}{dt} = -10(i-2) \quad \text{or} \quad \frac{d(i-2)}{i-2} = -10dt
\]

\( (i-2)|0 \text{ to } i-2 = e^{-10t} = i-2 \), thus \( i(t) = 2 + e^{-10t} \).
Differentiator Circuit

\[ v_o = -RC \frac{dv_{in}}{dt} \]

\[ i_{in} = \frac{dq}{dt} = C \frac{dv_{in}}{dt} = \frac{0-v_o}{R} \rightarrow v_o = -RC \frac{dv_{in}}{dt} \]
Op Amp Integrator

\[ i_{in} = \frac{v_{in}}{R} \quad v_c = \frac{q}{C} = \frac{1}{C} \int i_{in} \, dt = \frac{1}{RC} \int v_{in} \, dt \]

\[ -v_o - v_c = 0 \quad \Rightarrow \quad v_o = -v_c = \frac{-1}{RC} \int v_{in} \, dt \]
Analog Computing - Solving an O.D.E.

\[ a \frac{dy}{dt} + by = c \]
REVIST of Source-Free Series RLC Circuits

- The solution of the source-free series RLC circuit is called as the natural response of the circuit.
- The circuit is excited by the energy initially stored in the capacitor and inductor.

The 2nd order DE turns out to be

\[
\frac{d^2 i}{dt^2} + \frac{R}{L} \frac{di}{dt} + \frac{i}{LC} = 0
\]
Solution of the Complementary Equation (Natural Response)

\[
\frac{d^2 i(t)}{dt^2} + 2\alpha \frac{di(t)}{dt} + \omega_0^2 i(t) = 0
\]

Try \( i_c(t) = Ke^{st} \):

\[
s^2Ke^{st} + 2\alpha sKe^{st} + \omega_0^2 Ke^{st} = 0
\]

Factoring:

\[
(s^2 + 2\alpha s + \omega_0^2)Ke^{st} = 0
\]

Characteristic equation:

\[
s^2 + 2\alpha s + \omega_0^2 = 0
\]

Exponential is the indestructible function by differentiation or integration.
Solution of the Complementary Equation (Natural Response)

Quadratic Equation: \( ax^2 + bx + c = 0 \)

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

Characteristic Equation: \( s^2 + 2\alpha s + \omega_0^2 = 0 \)

\[
s = \frac{-2\alpha \pm \sqrt{(2\alpha)^2 - 4\omega_0^2}}{2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}
\]
Source-Free Series

There are three possible solutions to the following 2nd order differential equation (the complementary equation):

\[
\frac{d^2 i}{dt^2} + 2\alpha \frac{di}{dt} + \omega_0^2 i = 0
\]

1. If \( \alpha > \omega_0 \), over-damped case (\( \zeta > 1 \))

\[ i(t) = A_1 e^{\alpha t} + A_2 e^{\alpha t} \quad \text{where} \quad s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} \]

2. If \( \alpha = \omega_0 \), critical damped case (\( \zeta = 1 \))

\[ i(t) = (A_1 + A_2 t) e^{-\alpha t} \quad \text{where} \quad s_{1,2} = -\alpha \]

3. If \( \alpha < \omega_0 \), under-damped case (\( \zeta < 1 \))

\[ i(t) = e^{-\alpha t} (B_1 \cos \omega_d t + B_2 \sin \omega_d t) \quad \text{where} \quad \omega_d = \sqrt{\omega_0^2 - \alpha^2} \]

Solution of the Complementary Equation

Roots of the characteristic equation:

\[
s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} \\

s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}
\]

\[ \zeta = \frac{\alpha}{\omega_0} \quad \text{Dampening ratio} \]