Problem 1. In the circuit in the figure, let \( i_s = 50e^{-2t} \) mA and \( v_1(0) = 50 \) V, \( v_2(0) = 20 \) V. Determine (a) \( v_1(t) \) and \( v_2(t) \), and (b) the energy in each capacitor at \( t = 0.5 \) s. Round all answers to two decimal places.

Solution 1. (a) The current through the 12\( \mu \)F capacitor is \( i_s \), so the voltage is given by the relationship
\[
v_1(t) = v_1(0) + \frac{1}{C} \int_{\tau=0}^{\tau=t} i_s(\tau) d\tau
\]
Thus,
\[
7 = 50 + \frac{1}{12 \mu \text{F}} \int_{\tau=0}^{\tau=t} 0.050e^{-2\tau} \text{ A} d\tau
\]
\[
= 50 + \frac{1}{12 \mu \text{F}} \left[ \frac{0.050 \text{A}}{-2s^{-1}} e^{-2\tau} \right]_{\tau=0}^{\tau=t}
\]
\[
= 50 - \frac{50(10^{-3})}{-2(12)(10^{-6})} \left[ e^{-2t} - e^{0} \right] \text{ V}
\]
\[
= 50 - \frac{625}{3} \left[ e^{-2t} - 1 \right] \text{ V (1)}
\]

The voltage \( v_2(t) \) is found by first observing that the capacitors 10\( \mu \)F and 60\( \mu \)F are in parallel (and hence have the same voltage across them \( v_2(t) \)), so they form an equivalent capacitor of 70\( \mu \)F. Therefore, the current \( i_s \) goes through the equivalent capacitor with a voltage \( v_2(t) \) across it. So
\[
v_2(t) = v_2(0) + \frac{1}{70 \mu \text{F}} \int_{\tau=0}^{\tau=t} i_s(\tau) d\tau
\]
\[
= 20 - \frac{50(10^{-3})}{-2(70)(10^{-6})} \left[ e^{-2t} - e^{0} \right] \text{ V}
\]
\[
= 20 - \frac{2500}{7} \left[ e^{-2t} - 1 \right] \text{ V (2)}
\]
(b) The energy in a capacitor is found to be \( W_C(t) = 0.5Cv_C^2(t) \), so

\[
W_{12}(0.5) = 0.5(12)(10^{-6})(50 - \frac{625}{3}(e^{-1} - 1))^2 \ J = 1.19 \ J \tag{3}
\]

\[
W_{10}(0.5) = 0.5(10)(10^{-6})(20 - \frac{2500}{7}(e^{-1} - 1))^2 \ J = 0.65 \ J \tag{4}
\]

\[
W_{60}(0.5) = 0.5(60)(10^{-6})(20 - \frac{2500}{7}(e^{-1} - 1))^2 \ J = 3.89 \ J \tag{5}
\]

**Problem 2.** Find the equivalent inductance of the circuit in the figure. Round to two decimal places.

**Solution 2.** The first things to note is that inductors \( L_4 \) and \( L_5 \) are in parallel. Also, \( L_1 \) and \( L_2 \) are also in parallel. This is observed by where they are connected on both sides. Therefore,

\[
L_1||L_2 = 19 \Omega \tag{6}
\]

Since the parallel connection of \( L_1 \) and \( L_2 \) is in series in \( L_3 \), then

\[
L_A = L_1||L_2 + L_3 = 19 \Omega + 38 \Omega = 57 \Omega \tag{7}
\]

This inductor \( L_A \) is in parallel with \( L_4 \) and \( L_5 \). So

\[
L_{eq} = L_A||L_4||L_5 = \left( \frac{1}{57} + \frac{1}{19} \right)^{-1} = \left( \frac{4}{57} \right)^{-1} = 14.25 \text{H} \tag{8}
\]

**Problem 3.** Determine the equivalent inductance \( L_{eq} \) for the following circuit.
Solution 3. The current \( i \) is the current through the 9H inductor, and has voltage \( v_1 = 9 \frac{di}{dt} \). Call the current through the 3H inductor \( i_2 \). Then by KCL, the current \( i_3 \) through the 5H inductor is

\[
i_3 = i - i_2
\]  

(9)

Therefore, since \( v_2 = 3 \frac{di_2}{dt} \), and the voltage in the 5H inductor is \( v_5 = 5 \frac{di_3}{dt} \), by KVL we have

\[
0 = 3 \frac{di_2}{dt} - 4 \frac{di}{dt} - 5 \frac{di_3}{dt}
\]

\[
= 3 \frac{di_2}{dt} - 4 \frac{di}{dt} - 5 \frac{d(i - i_2)}{dt}
\]

\[
= 3 \frac{di_2}{dt} - 4 \frac{di}{dt} - 5 \frac{di}{dt} + 5 \frac{di_2}{dt}
\]

\[
\Rightarrow \frac{di_2}{dt} = \frac{9}{8} \frac{di}{dt}
\]  

(10)

Now, we use KVL around the loop to note that \( 0 = V_a - v_1 - v_2 \). Also, remember that the voltage across this inductive circuit has a voltage across the equivalent inductance as \( V_a b = L_{eq} \frac{di}{dt} \), since current \( i \) is the one going into the equivalent inductance. So

\[
L_{eq} \frac{di}{dt} = 9 \frac{di}{dt} + 3 \cdot \frac{9}{8} \frac{di}{dt}
\]

\[
= 12.625 \frac{di}{dt}
\]

\[
\Rightarrow L_{eq} = 12.625 \text{ H}
\]  

(11)

Problem 4. Lab has many 20\( \mu \)F capacitors rated at 250V. You need a 80\( \mu \)F capacitor rated at 500V. How many 20\( \mu \)F capacitors are needed and how should you connect them.

Solution 4. Since each capacitor is rated at 250 V, then two capacitors in series have an equivalent rating of 500V. However, the two 20\( \mu \)F capacitors in series now have an equivalent capacitance of 10\( \mu \)F. So you need 8 of these 10\( \mu \)F equivalent capacitors in parallel to get 80\( \mu \)F with a 500V rating.

Problem 5. Find the voltage across the capacitors in the circuit under DC conditions.
Solution 5. Since the voltage supply is DC, the current is also DC. Hence, the current will not pass through the capacitors, i.e. they are open circuits, and have voltages across them equal to any connection that is parallel to them. So the voltage $v_1$ is the voltage across the 20Ω resistor, and the voltage $v_2$ is the source 160V minus the voltages across the 70 and 50 Ω resistors. Therefore, we can do DC analysis on the resistor circuit.

KCL (no current through 70 ohm resistor)

$$0 = \frac{160 - V}{50} - \frac{V}{10 + 20}$$

$$\Rightarrow V = 60 \text{ V} \quad (12)$$

The voltage across the 20 ohm resistor is $v_1 = V \frac{20}{30} = 40$V. The voltage $v_2 = V = 60$, so

$$v_1 = 40 \text{V} \quad \text{and} \quad v_2 = 60 \text{V} \quad (13)$$