Lecture 2
Fundamentals of Electrical Engineering

Introduction

1. Define current, voltage, and power, including their units.

2. Calculate power and energy, as well as determine whether energy is supplied or absorbed by a circuit element.

3. State and apply basic circuit laws.

4. Solve for currents, voltages, and powers in simple circuits.
Electrical Current

Electrical current is the time rate of flow of electrical charge through a conductor or circuit element. The units are amperes (A), which are equivalent to coulombs per second (C/s).

\[ i(t) = \frac{dq(t)}{dt} \]
\[ q(t) = \int_{t_0}^{t} i(t)\,dt + q(t_0) \]
Dissipates energy in the form of heat

Stores energy in a magnetic field

Stores energy in an electric field

One example is a battery that stores energy in the form of an electrochemical reaction

Figure 1.3 An electrical circuit consists of circuit elements, such as voltage sources, resistances, inductances, and capacitances, connected in closed paths by conductors.

Figure 1.6 In analyzing circuits, we frequently start by assigning current variables $i_1$, $i_2$, $i_3$, and so forth.
Direct Current
Alternating Current

When a current is constant with time, we say that we have **direct current**, abbreviated as dc. On the other hand, a current that varies with time, reversing direction periodically, is called **alternating current**, abbreviated as ac.
Figure 1.8 Ac currents can have various waveforms.

(a) Triangular waveform  (b) Square waveform

Figure 1.9 Reference directions can be indicated by labeling the ends of circuit elements and using double subscripts on current variables. The reference direction for $i_{ab}$ points from $a$ to $b$. On the other hand, the reference direction for $i_{ba}$ points from $b$ to $a$. 
Voltages

The **voltage** associated with a circuit element is the energy transferred per unit of charge that flows through the element. The units of voltage are volts (V), which are equivalent to joules per coulomb (J/C).

*Figure 1.10*  Energy is transferred when charge flows through an element having a voltage across it.
Figure 1.12 The voltage $v_{ab}$ has a reference polarity that is positive at point $a$ and negative at point $b$.

Figure 1.13 The positive reference for $v$ is at the head of the arrow.
Power and Energy

\[ p(t) = v(t)i(t) \]  \hspace{1cm} \text{Watts}

\[ w = \int_{t_1}^{t_2} p(t)dt \]  \hspace{1cm} \text{Joules}

Current is flowing in the passive configuration. Energy is being absorbed by the element (e.g. a resistor)

If the current flows opposite to the passive configuration, the power is given by \( p = -vi \)

Energy is being provided by the element (e.g. a battery)
\[ P_a = i_a v_a = (2A) \cdot (12V) = 24W, \text{ energy is being absorbed} \]
\[ P_b = -i_b v_b = -(1A) \cdot (12V) = -12W, \text{ energy is being supplied} \]
\[ P_c = i_c v_c = (-3A) \cdot (12V) = -36W, \text{ energy is being supplied} \]

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**Power and Energy**

\[ v(t) = 12 \text{ V} \]
\[ i(t) = 2e^{t} \text{ A} \]
\[ p(t) = v(t)i(t) = 24e^{-t} \]

\[ w = \int_{0}^{\infty} p(t)dt = \int_{0}^{\infty} 24e^{-t} dt = -24e^{-\infty} - (-24e^{0}) = 24J \]
An example would be a battery.

An example would be a transformer.

\( v_x = 12 \text{ V} \) independent of the load

\( v_x = 0 \)
Figure 1.32 Dependent voltage sources (also known as controlled voltage sources) are represented by diamond-shaped symbols. The voltage across a controlled voltage source depends on a current or voltage that appears elsewhere in the circuit.

Figure 1.33 Independent current sources.

(a) DC current source

(b) AC current source
Resistors and Ohm’s Law

\[ v = iR \]

\[ v_{ab} = i_{ab}R \]

The units of resistance are Volts/Amp which are called “ohms”. The symbol for ohms is omega: \( \Omega \)
George Simon Ohm, 1789-1854

In 1805 Ohm entered the University of Erlangen but he became rather carried away with student life. Rather than concentrate on his studies he spent much time dancing, ice skating and playing billiards.

http://www-groups.dcs.st-and.ac.uk/~history/Mathematicians/Ohm.html

Conductance

\[ G = \frac{1}{R} \]

\[ i = Gv \]

The units of conductance are \(1/\Omega\) or \(\Omega^{-1}\). The units are called “siemens”
Resistance Related to Physical Parameters

\[ R = \frac{\rho L}{A} \]

\( \rho \) is the resistivity of the material used to fabricate the resistor. The units of resistivity are ohm-meters (\( \Omega \)-m)

Power dissipation in a resistor

\[ p = vi = Ri^2 = \frac{v^2}{R} \]
KIRCHHOFF’S CURRENT LAW

• The net current entering a node is zero.

• Alternatively, the sum of the currents entering a node equals the sum of the currents leaving a node.

Gustav Kirchhoff (1824-1887)

Kirchhoff's Current Law

The principle of conservation of electric charge implies that:

At any point in an electrical circuit where charge density is not changing in time, the sum of currents flowing towards that point is equal to the sum of currents flowing away from that point.

A charge density changing in time would mean the accumulation of a net positive or negative charge, which typically cannot happen to any significant degree because of the strength of electrostatic forces: the charge buildup would cause repulsive forces to disperse the charges.

http://www-groups.dcs.st-and.ac.uk/~history/Mathematicians/Kirchhoff.html
(a) Currents into the node = 1A + 3A = 4A current out of the node $i_a$.

(b) Currents into the node = 3A + 1A + $i_b$ = current out of the node = 2A so $i_b$ = 2A - 4A = -2A.

(c) Currents into the node = 1A + $i_c$ + 3A + 4A = current out of the node = 0 amps so $i_c$ = -8A.

Series Connection

Figure 1.20 Elements $A$, $B$, and $C$ are connected in series.
Which elements are in series?

KIRCHHOFF’S VOLTAGE LAW

The algebraic sum of the voltages equals zero for any closed path (loop) in an electrical circuit.
Loop 1: \(-v_a + v_b + v_c = 0\)

Loop 2: \(-v_c - v_d + v_e = 0\)

Loop 3: \(v_a - v_c + v_d - v_b = 0\)

Figure 1.23 In applying KVL to a loop, voltages are added or subtracted depending on their reference polarities relative to the direction of travel around the loop.

Moving from + to – we add \(v_a\).

Moving from – to + we subtract \(v_a\).
Parallel Connection

KVL through A and B: \(-v_a + v_b = 0 \rightarrow v_a = v_b\)

KVL through A and C: \(-v_a - v_c = 0 \rightarrow v_a = -v_c\)

Figure 1.27 For this circuit, we can show that \(v_a = v_b = -v_c\). Thus the magnitudes and actual polarities of all three voltages are the same.

Figure 1.26 In this circuit, elements A and B are in parallel. Elements D, E, and F form another parallel combination.
\[-3V - 5V + v_c = 0 \rightarrow v_c = 8V\]
\[-8V - (-10V) + v_e = 0 \rightarrow v_e = -2V\]

Which elements are in series? Which elements are in parallel?

Find the current, voltage and power for each element:

(a) Circuit diagram

(b) KVL requires that \(v_R = 10\ V\)

(c) Ohm’s law yields \(i_R = v_R / R = 2\ A\)

(d) KCL requires that \(i_s = i_R\)
Power

\[ P_R = i_R v_R = (2A)(10V) = 20W \text{ (power dissipated)} \]

\[ = i_R^2 R \]

\[ = V_R^2 / R \]

\[ P_S = -v_S i_S = -(10V)(2A) = -20W \text{ (power supplied)} \]

\[ P_R + P_S = 0 \]

Example

- What is the current \( i_R \) flowing through the resistor?
- What is the power for each element in the circuit?
- Which elements are absorbing power?
Since all of the elements are in series, the same current $i_R = 2A$ runs through each of them.

The voltage drop across the resistor:

$$v_R = iR = (2A)(5\Omega) = 10V$$

Apply KVL:

$$v_c = v_R + 10 = 20V$$

The power dissipated in each element:

$$p_R = iR^2 = (2A)^2(5\Omega) = 20W \text{ (absorbing)}$$

$$p_{vs} = iv = (2A)(10V) = 20W \text{ (absorbing)}$$

$$p_{cs} = -iv = -(2A)(20V) = 40W \text{ (supplying)}$$

Example

Use Ohm’s law, KVL and KCL to find $V_x$. 

\[ \text{Diagram of circuit with 2A current source and 5\Omega, 10\Omega, and 5\Omega resistors.} \]
Ohm’s Law:

\[ V_{R4} = (1A)(5\Omega) = 5V = V_{R2} = V_{R3} \]

\[ I_{R2} = \frac{5V}{10\Omega} = 0.5A \]

\[ I_{R3} = \frac{5V}{5\Omega} = 1A \]

KCL: \[ I_T = I_{R2} + I_{R3} + I_{R4} = 0.5A + 1A + 1A = 2.5A \]

Ohm’s Law: \[ V_{R1} = I_T R_1 = (2.5A)(5\Omega) = 12.5V \]

KVL: \[ V_x = V_{R1} + V_{R4} = 12.5V + 5V = 17.5V \]