Lecture 3
Circuit Laws,
Voltage & Current Dividers
**KIRCHHOFF’S CURRENT LAW**

- *The net current entering a node is zero.*

- *Alternatively, the sum of the currents entering a node equals the sum of the currents leaving a node.*
Figure 1.20  Elements A, B, and C are connected in series.
KIRCHHOFF’S VOLTAGE LAW

The algebraic sum of the voltages equals zero for any closed path (loop) in an electrical circuit.
Figure 1.27 For this circuit, we can show that $v_a = v_b = -v_c$. Thus the magnitudes and actual polarities of all three voltages are the same.
Using KVL, KCL, and Ohm’s Law to Solve a Circuit
\[ i_y = \frac{15 \text{ V}}{5 \Omega} = 3 \text{ A} \]

\[ i_x + 0.5i_x = i_y \]

\[ i_x = 2 \text{ A} \]
\[ i_x = 2 \text{ A} \]

\[ v_x = 10i_x = 20 \text{ V} \]

\[ V_s = v_x + 15 \]

\[ V_s = 35 \text{ V} \]
Chapter 2
Resistive Circuits

1. Solve circuits (i.e., find currents and voltages of interest) by combining resistances in series and parallel.

2. Apply the voltage-division and current-division principles.

3. Solve circuits by the node-voltage technique.
4. Solve circuits by the mesh-current technique.

5. Find Thévenin and Norton equivalents and apply source transformations.

6. Apply the superposition principle.

7. Draw the circuit diagram and state the principles of operation for the Wheatstone bridge.
\[ v = v_1 + v_2 + v_3 = iR_1 + iR_2 + iR_3 = i(R_1 + R_2 + R_3) = iR_{eq} \]

**Figure 2.1** Series resistances can be combined into an equivalent resistance.
(a) Three resistances in parallel

(b) Equivalent resistance

Figure 2.2 Parallel resistances can be combined into an equivalent resistance.

\[ i = i_1 + i_2 + i_3 = \frac{v}{R_1} + \frac{v}{R_2} + \frac{v}{R_3} = v \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right) = \frac{v}{R_{eq}} \]
Figure 2.3 Resistive network for Example 2.1.
Figure 2.4 Resistive networks for Exercise 2.1.
\[ R_1 = 2 \, \Omega \]

\[ R_2 = 6 \, \Omega \quad R_3 = 3 \, \Omega \quad R_4 = 2 \, \Omega \]

\[
\frac{1}{R_{eq}} = \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} = \frac{1}{6} + \frac{1}{3} + \frac{1}{2} = \frac{1}{6} + \frac{2}{6} + \frac{3}{6} = 1
\]
\[ R_{total} = R_1 + 1\Omega = 3\Omega \]
\[
\frac{1}{R_{eq}} = \frac{1}{3} + \frac{1}{6} = \frac{2}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}
\]

\[R_{eq} = 2.0 \Omega\]
\[ R_{eq} = 8\Omega + 2\Omega = 10\Omega \]
\[
\frac{1}{R_{eq}} = \frac{1}{10} + \frac{1}{10} = \frac{2}{10} \Omega \rightarrow R_{eq} = 5\Omega
\]
\( R_1 = 100 \, \Omega \)

\( R_2 = 50 \, \Omega \)

\( R_3 = 75 \, \Omega \)

\( R_4 = 25 \, \Omega \)
The diagram shows a complex electrical circuit with labeled resistors.

- \( R_1 = 1 \, \text{k}\Omega \)
- \( R_2 = 2 \, \text{k}\Omega \)
- \( R_3 = 3 \, \text{k}\Omega \)
Circuit Analysis using Series/Parallel Equivalents

1. Begin by locating a combination of resistances that are in series or parallel. Often the place to start is farthest from the source.

2. Redraw the circuit with the equivalent resistance for the combination found in step 1.
3. Repeat steps 1 and 2 until the circuit is reduced as far as possible. Often (but not always) we end up with a single source and a single resistance.

4. Solve for the currents and voltages in the final equivalent circuit.
Figure 2.5 A circuit and its simplified versions. See Example 2.2.
Figure 2.6 After reducing the circuit to a source and an equivalent resistance, we solve the simplified circuit. Then, we transfer results back to the original circuit. Notice that the logical flow in solving for currents and voltages starts from the simplified circuit in (c).
\[ i_2 = \frac{v_2}{R_2} = \frac{60V}{30\Omega} = 2A \]

\[ i_3 = \frac{v_2}{R_3} = \frac{60V}{60\Omega} = 1A \]

\[ v_1 = R_1i_1 = (10\Omega)(3A) = 30V \]
Figure 2.7 Circuits for Exercise 2.2.
\[ I_{R_{eq}} = \frac{1}{1 + \frac{1}{R_2} + \frac{1}{R_3}} = \frac{1}{\frac{1}{120} + \frac{1}{120}} = 9.2 \text{ A} \]

\[ R_{eq} = \frac{120}{13} = 9.2 \]

\[ V_{i1} = \frac{20}{1.04} = 19.2 \text{ V} \]

\[ V_1 = 1.04 \text{ A} \times 19.2 \text{ V} = 19.2 \text{ V} \]

\[ V_1 = (1.04 \text{ A} \times 9.2 \Omega) = 9.6 \text{ W} \]

\[ I_1 = 9.6 \text{ V} / 19.2 \text{ A} = 0.5 \text{ A} \]

\[ I_2 = 9.6 \text{ V} / 9.2 \Omega = 1.04 \text{ A} \]
\[ V_1 \times (2\, \text{amp})(1-\text{ohm}) = 20\, \text{V} \]

\[ i_1 = \frac{20\, \text{V}}{20\, \text{ohm}} = 1\, \text{amp} \]

\[ V_2 \times 10\, \text{ohm} = 100\, \text{Vlit} \]

\[ V_2 = 10\, \text{Vlit} \times 10\, \text{ohm} = 100\, \text{Vlit} \]

\[ i_2 = \frac{V}{20\, \text{ohm}} = 5\, \text{amp} \]

\[ v_3 = (15\, \text{amp})(0.3\, \text{ohm}) = 4.5\, \text{Wlit} \]

\[ V_3 = 25\, \text{Vlit} - 0.1\, \text{Vlit} = 24.9\, \text{Vlit} \]
Figure 2.8 Circuit used to derive the voltage-division principle.
Voltage Division

Of the total voltage, the fraction that appears across a given resistance in a series circuit is the ratio of the given resistance to the total series resistance.

\[ v_1 = R_1 i = \frac{R_1}{R_1 + R_2 + R_3} v_{\text{total}} \]

\[ v_2 = R_2 i = \frac{R_2}{R_1 + R_2 + R_3} v_{\text{total}} \]
Figure 2.9 Circuit for Example 2.3.
Application of the Voltage-Division Principle

\[ v_1 = \frac{R_1}{R_1 + R_2 + R_3 + R_4} \times \frac{v_{\text{total}}}{v_{\text{total}}} \]

\[ = \frac{1000}{1000 + 1000 + 2000 + 6000} \times 15 \]

\[ = 1.5V \]
Figure 2.10 Circuit used to derive the current-division principle.

\[ R_{eq} = \frac{R_1 R_2}{R_1 + R_2} \rightarrow v = R_{eq} i_{total} = \frac{R_1 R_2}{R_1 + R_2} i_{total} \]
Current Division

For two resistances in parallel, the fraction of the total current flowing in a resistance is the ratio of the other resistance to the sum of the two resistances.

\[
i_1 = \frac{v}{R_1} = \frac{R_2}{R_1 + R_2} i_{\text{total}}
\]

\[
i_2 = \frac{v}{R_2} = \frac{R_1}{R_1 + R_2} i_{\text{total}}
\]
Find $i_3$

(a) Original circuit

(b) Equivalent circuit obtained by combining $R_2$ and $R_3$

\[
\begin{align*}
    i_s &= \frac{v_s}{R_1 + R_x} = \frac{100}{60 + 20} = 1.25 A \\
    i_3 &= \frac{R_2}{R_2 + R_3} i_s = \frac{30}{30 + 60} (1.25) = 0.417 A
\end{align*}
\]
Find $i_1$

(a) Original circuit

(b) Circuit after combining $R_2$ and $R_3$

Figure 2.12 Circuit for Example 2.5.
Application of the Current-Division Principle

(a) Original circuit

(b) Circuit after combining $R_2$ and $R_3$

\[
R_{eq} = \frac{R_2 R_3}{R_2 + R_3} = \frac{30 \times 60}{30 + 60} = 20 \Omega
\]

\[
i_1 = \frac{R_{eq}}{R_1 + R_{eq}} i_s = \frac{20}{10 + 20} \times 15 = 10 \text{A}
\]
Use the voltage division principle to find the unknown voltages.

\[ v_s = 120 \text{ V} \]

\[ R_1 = 5 \Omega \]

\[ R_2 = 10 \Omega \]

\[ R_3 = 15 \Omega \]

\[ R_4 = 30 \Omega \]

\[ v_s = 20 \text{ V} \]

\[ R_1 = 3 \Omega \]

\[ R_2 = 7 \Omega \]

\[ R_3 = 5 \Omega \]

\[ R_4 = 4 \Omega \]

\[(a)\quad \text{and} \quad (b)\]

**Figure 2.14** Circuits for Exercise 2.3.
\[ v_1 = \frac{R_1}{R_1 + R_2 + R_3 + R_4} \quad v_s = \frac{5}{5+10+15+30} (120V) = \frac{120V}{12} = 10V \]

\[ v_2 = \frac{R_2}{R_1 + R_2 + R_3 + R_4} \quad v_s = \frac{10}{5+10+15+30} (120V) = \frac{120V}{6} = 20V \]

\[ v_3 = \frac{R_3}{R_1 + R_2 + R_3 + R_4} \quad v_s = \frac{15}{5+10+15+30} (120V) = \frac{120V}{4} = 30V \]

\[ v_4 = \frac{R_4}{R_1 + R_2 + R_3 + R_4} \quad v_s = \frac{30}{5+10+15+30} (120V) = \frac{120V}{2} = 60V \]
\[ R_{eq} = \frac{1}{\frac{1}{7} + \frac{1}{5}} = 2.92\Omega \]

\[ v_1 = \frac{R_1}{R_1 + R_{eq} + R_4} v_s = \frac{3}{3 + 2.92 + 4}(20V) = \frac{(3)(20V)}{9.92} = 6.05V \]

\[ v_2 = \frac{R_{eq}}{R_1 + R_{eq} + R_4} v_s = \frac{2.92}{3 + 2.92 + 4}(20V) = \frac{(2.92)(20V)}{9.92} = 5.89V \]

\[ v_4 = \frac{R_4}{R_1 + R_{eq} + R_4} v_s = \frac{4}{3 + 2.92 + 4}(20V) = \frac{(4)(20V)}{9.92} = 8.06V \]
Use the current division principle to find the currents

**Figure 2.15** Circuits for Exercise 2.4.
\[ i_1 = \frac{R_3}{R_1 + R_2 + R_3} \quad i_t = \frac{15}{10 + 20 + 15} (3) = 1A \]

\[ i_3 = \frac{R_1 + R_2}{R_1 + R_2 + R_3} \quad i_t = \frac{30}{10 + 20 + 15} (3) = 2A \]
\[ R_{eq} = \frac{1}{\frac{1}{10} + \frac{1}{10}} = 5 \]

\[ i_s = 3 \text{ A} \]

\[ i_1 = \frac{R_{eq}}{R_{eq} + R_1} i_s = \frac{5}{5 + 10} (3) = 1 \text{ A} \]

\[ i_2 = \frac{R_{eq}}{R_{eq} + R_1} i_s = \frac{5}{5 + 10} (3) = 1 \text{ A} \]

\[ i_3 = \frac{R_{eq}}{R_{eq} + R_1} i_s = \frac{5}{5 + 10} (3) = 1 \text{ A} \]
Resistor Cube

All resistors: 1 Ω