Lecture 4
Node/Loop Analysis
Although they are very important concepts, series/parallel equivalents and the current/voltage division principles are not sufficient to solve all circuits.
Node Voltage Analysis

The first step in node analysis is to select a reference node and label the voltages at each of the other nodes.
Node Voltage Analysis

\[ v_1 = v_s \]

\[ \frac{v_2 - v_1}{R_2} + \frac{v_2}{R_4} + \frac{v_2 - v_3}{R_3} = 0 \]

\[ \frac{v_3 - v_1}{R_1} + \frac{v_3}{R_5} + \frac{v_3 - v_2}{R_3} = 0 \]
Example 2.6

\[
\frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2} + i_s = 0
\]

\[
\frac{v_2 - v_1}{R_2} + \frac{v_2}{R_3} + \frac{v_2 - v_3}{R_4} = 0
\]

\[
\frac{v_3}{R_5} + \frac{v_3 - v_2}{R_4} = i_s
\]
Exercise 2.6
\[
\begin{align*}
\frac{v_1 - v_2}{R_2} + \frac{v_1 - v_3}{R_1} &= i_a \\
\frac{v_2 - v_1}{R_2} + \frac{v_2}{R_3} + \frac{v_2 - v_3}{R_4} &= 0 \\
\frac{v_3 - v_2}{R_4} + \frac{v_3 - v_1}{R_1} + \frac{v_3}{R_5} + i_b &= 0
\end{align*}
\]
Exercise 2.8
Exercise 2.8

\[
\frac{V_1 - 10}{2} + \frac{V_1}{5} + \frac{V_1 - V_2}{10} = 0 \rightarrow 5V_1 - 50 + 2V_1 + V_1 - V_2 = 0 \rightarrow 8V_1 - V_2 = 50
\]

\[
\frac{V_2 - 10}{10} + \frac{V_2 - V_1}{10} + \frac{V_2}{5} = 0 \rightarrow V_2 - 10 + V_2 - V_1 + 2V_2 = 0 \rightarrow -V_1 + 4V_2 = 10
\]

\[
8V_1 - V_2 = 50
\]

\[-8V_1 + 32V_2 = 80
\]

\[3V_2 = 130
\]

\[V_2 = \frac{130}{31} = 4.19
\]

\[V_1 = 4V_2 - 10 = 6.77
\]
Exercise 2.9
Exercise 2.9

\[ \frac{v_1 - v_3}{20} + \frac{v_1}{5} + \frac{v_1 - v_2}{10} = 0 \rightarrow v_1 - v_3 + 4v_1 + 2v_1 - 2v_2 = 0 \]

\[ \frac{v_2 - v_1}{10} + \frac{v_2 - v_3}{5} + 10 = 0 \rightarrow v_2 - v_1 + 2v_2 - 2v_3 + 100 = 0 \]

\[ \frac{v_3}{10} + \frac{v_3 - v_1}{20} + \frac{v_3 - v_2}{5} = 0 \rightarrow 2v_3 + v_3 - v_1 + 4v_3 - 4v_2 = 0 \]

\[ 7v_1 - 2v_2 - v_3 = 0 \]

\[ -v_1 + 3v_2 - 2v_3 = -100 \]

\[ -v_1 - 4v_2 + 7v_3 = 0 \]
Figure 2.24 A supernode is formed by drawing a dashed line enclosing several nodes and any elements connected between them.
Circuits with Voltage Sources

We obtain dependent equations if we use all of the nodes in a network to write KCL equations.
\[
\frac{v_1}{R_2} + \frac{v_1 - (-15)}{R_1} + \frac{v_2}{R_4} + \frac{v_2 - (-15)}{R_3} = 0
\]
Use KVL for a second independent equation:

\[ v_2 - v_1 = 10V \]
Exercise 2.11
KVL: \(-v_1 + 10 + v_2 = 0\)

KCL for supernode at 10V source:
\[
\frac{v_1}{R_1} + \frac{v_1 - v_3}{R_2} + \frac{v_2 - v_3}{R_3} = 1
\]

KCL for node 3:
\[
\frac{v_3 - v_1}{R_2} + \frac{v_3 - v_2}{R_3} + \frac{v_3}{R_4} = 0
\]

KCL for reference node:
\[
\frac{v_1}{R_1} + \frac{v_3}{R_4} = 1
\]
Node-Voltage Analysis with a Dependent Source

First, we write KCL equations at each node, including the current of the controlled source just as if it were an ordinary current source.
Example 2.9

\[
\frac{v_1 - v_2}{R_1} = i_s + 2i_x \\
\frac{v_2 - v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_2 - v_3}{R_3} = 0 \\
\frac{v_3 - v_2}{R_3} + \frac{v_3}{R_4} + 2i_x = 0
\]
Example 2.9

Next, we find an expression for the controlling variable $i_x$ in terms of the node voltages.

\[ i_x = \frac{v_3 - v_2}{R_3} \]
Example 2.9

Substitution yields:

\[
\frac{v_1 - v_2}{R_1} = i_s + 2 \frac{v_3 - v_2}{R_3}
\]

\[
\frac{v_2 - v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_2 - v_3}{R_3} = 0
\]

\[
\frac{v_3 - v_2}{R_3} + \frac{v_3}{R_4} + 2 \frac{v_3 - v_2}{R_3} = 0
\]
Node-Voltage Analysis

1. Select a reference node and assign variables for the unknown node voltages. If the reference node is chosen at one end of an independent voltage source, one node voltage is known at the start, and fewer need to be computed.
2. Write network equations. First, use KCL to write current equations for nodes and supernodes. Write as many current equations as you can without using all of the nodes. Then if you do not have enough equations because of voltage sources connected between nodes, use KVL to write additional equations.
3. If the circuit contains dependent sources, find expressions for the controlling variables in terms of the node voltages. Substitute into the network equations, and obtain equations having only the node voltages as unknowns.
4. Put the equations into standard form and solve for the node voltages.

5. Use the values found for the node voltages to calculate any other currents or voltages of interest.
Exercise 2.12(a)

\[ v_2 - v_1 = 10V \rightarrow v_2 = 10 + v_1 \]

\[ \frac{v_1}{10} + \frac{v_2}{5} = 1 \rightarrow v_1 + 2(10 + v_1) = 10 \]

\[ 3v_1 = -10 \]

\[ v_1 = -\frac{10}{3} \rightarrow v_2 = \frac{20}{3} \]

\[ i_a = \frac{v_2}{5} = \frac{4}{3} = 1.33A \]
Exercise 2.12(b)

\[
\frac{v_1 - 25}{10} + \frac{v_1}{10} + \frac{v_1 - v_2}{20} = 0
\]
\[
\frac{v_2}{20} + \frac{v_2 - v_1}{20} + \frac{v_2 - 25}{5} = 0
\]

\[v_1 = 13.79\text{V}\]
\[v_2 = 18.97\text{V}\]

\[
ib = \frac{v_1 - v_2}{20} = -0.259\text{A}
\]
Exercise 2.13(a)

\[
\frac{v_1 - 10}{5} + \frac{v_1}{5} = 2i_x \rightarrow 2v_1 - 10 = 10i_x
\]

\[
i_x = \frac{10 - v_1}{5} \rightarrow -2v_1 + 20 = 10i_x
\]

\[
20i_x = 10
\]

\[
i_x = 0.5 A
\]
Exercise 2.13(b)

\[
\frac{v_1}{5} + \frac{v_1 - 2i_y}{2} + 3 = 0
\]

\[
\frac{v_2}{5} + \frac{v_2 - 2i_y}{10} - 3 = 0
\]

\[i_y = \frac{v_2}{5} \rightarrow i_y = 2.31A\]