Lecture 6

Thévenin Equivalent Circuits
Thévenin Equivalent Circuits

Figure 2.40 A two-terminal circuit consisting of resistances and sources can be replaced by a Thévenin equivalent circuit.
Thévenin Equivalent Circuits

Figure 2.41 Thévenin equivalent circuit with open-circuited terminals. The open-circuit voltage $v_{oc}$ is equal to the Thévenin voltage $V_t$.

$$V_t = v_{oc}$$
Thévenin Equivalent Circuits

Figure 2.42 Thévenin equivalent circuit with short-circuited terminals. The short-circuit current is $i_{sc} = \frac{V_t}{R_t}$.

$$i_{sc} = \frac{V_t}{R_t}$$
Thévenin Equivalent Circuits

\[ R_t = \frac{V_{oc}}{i_{sc}} \]
Find the Thevenin Equivalent Circuit

(a) Original circuit

(b) Analysis with an open circuit

(c) Analysis with a short circuit

(d) Thévenin equivalent

Figure 2.43 Circuit for Example 2.14.
Find the Thevenin Equivalent Circuit

(a) Original circuit

(b) Analysis with an open circuit

Use voltage divider formed by $R_1$ and $R_2$ to find $v_{oc}$:

\[
v_{oc} = \frac{R_2}{R_1 + R_2} v_s = \frac{50}{100 + 50} 15V = 5V
\]
Find the Thevenin Equivalent Circuit

\[ i_{sc} = \frac{v_s}{R_1} = \frac{15V}{100\Omega} = 0.15A \]
Find the Thevenin Equivalent Circuit

\[ R_t = 5 \Omega \]

\[ V_t = 5 \text{ V} \]

\[ R_t = \frac{V_{oc}}{i_{sc}} = \frac{5V}{0.15A} = 33.3 \Omega \]
Find the Thevenin Equivalent Circuit

\[ V_{oc} = (5A)(10\Omega) = 50V \]

*Figure 2.44* Circuit for Exercise 2.22.
Find the Thevenin Equivalent Circuit

Use current divider to find $i_{sc}$:

$$i_{sc} = \frac{R_1}{R_1 + R_2} \times 5A = \frac{10}{10 + 40} \times 5A = 1A$$

$$R_t = \frac{V_{oc}}{i_{sc}} = \frac{50V}{1A} = 50\Omega$$
Find the Thevenin Equivalent Circuit

\[ R_t = 50 \Omega \]

\[ V_t = 50 \text{V} \]
Finding the Thévenin Resistance Directly

We can find the Thévenin resistance by zeroing the sources in the original network and then computing the resistance between the terminals.

When zeroing a voltage source, it becomes a short circuit. When zeroing a current source, it becomes an open circuit.
Figure 2.45 When the source is zeroed, the resistance seen from the circuit terminals is equal to the Thévenin resistance.
Find the Thevenin Equivalent

(a) Original circuit

\[ v_s = 20 \text{ V} \]

\[ R_1 = 5 \Omega \]

\[ R_2 = \frac{20 \Omega}{2} \]

\[ 2 \text{ A} \]

(b) Circuit with sources zeroed

\[ R_1 = 5 \Omega \]

\[ R_2 = \frac{20 \Omega}{2} \]

\[ \rightarrow R_{eq} = R_t \]

(c) Circuit with a short circuit

\[ v_s = 20 \text{ V} \]

\[ R_1 = 5 \Omega \]

\[ i_1 \]

\[ i_2 \]

\[ i_{sc} \]

\[ R_2 = \frac{20 \Omega}{2} \]

\[ 2 \text{ A} \]

(d) Thévenin equivalent circuit

\[ V_t = 24 \text{ V} \]

\[ R_t = 4 \Omega \]

Figure 2.46 Circuit for Example 2.15.
Find the Thevenin Equivalent

\[ R_{eq} = R_t = \frac{R_1 R_2}{R_1 + R_2} = \frac{100}{25} = 4 \Omega \]
Find the Thevenin Equivalent

\[ i_2 = 0 \]

\[ i_1 = \frac{20V}{5\Omega} = 4A \]

\[ i_{sc} = i_1 + 2A = 4A + 2A = 6A \]
Find the Thevenin Equivalent

\[ V_t = R_t i_{sc} = (4\Omega)(6A) = 24\text{V} \]
Find the Thevenin Resistance

Figure 2.47 Circuits for Exercise 2.24.
Find the Thevenin Resistance

Zero the voltage source

\[ R_t = 10 + \frac{(5)(20)}{5 + 20} = 10 + 4 = 14\Omega \]
Find the Thevenin Resistance

Zero the current source

\[ R_t = 10 + 20 = 30\Omega \]
Find the Thevenin Resistance

Zero the current and voltage sources
Find the Thevenin Resistance
Figure 2.48 Circuit for Example 2.16.
Thevenin Equivalent with a Dependent Source

(b) Circuit with an open circuit

\[ KCL \text{ at Node 1: } i_x + 2i_x = \frac{v_{oc}}{10} = 3i_x \]

\[ i_x = \frac{10 - v_{oc}}{5} \rightarrow 3 \frac{10 - v_{oc}}{5} = \frac{v_{oc}}{10} \rightarrow v_{oc} = 8.57V \]
Thevenin Equivalent with a Dependent Source

(c) Circuit with a short circuit

\[ i_x + 2i_x = 3i_x = i_{sc} \]

\[ i_x = \frac{10V}{5\Omega} = 2A \rightarrow i_{sc} = 6A \]

\[ R_t = \frac{V_t}{i_{sc}} = \frac{8.57V}{6A} = 1.43\Omega \]
Figure 2.49 The Norton equivalent circuit consists of an independent current source $I_n$ in parallel with the Thévenin resistance $R_t$. 
Figure 2.50 The Norton equivalent circuit with a short circuit across its terminals.

\[ I_n = i_{sc} \]
Step-by-step
Thévenin/Norton-Equivalent-Circuit Analysis

1. Perform two of these:
   a. Determine the open-circuit voltage \( V_t = v_{oc} \).
   b. Determine the short-circuit current \( I_n = i_{sc} \).
   c. Zero the sources and find the Thévenin resistance \( R_t \) looking back into the terminals.
2. Use the equation \( V_t = R_t \, I_n \) to compute the remaining value.

3. The Thévenin equivalent consists of a voltage source \( V_t \) in series with \( R_t \).

4. The Norton equivalent consists of a current source \( I_n \) in parallel with \( R_t \).