Lecture 8
Amplifiers
Goals

1. Use various amplifier models to calculate amplifier performance for given sources and loads.

2. Compute amplifier efficiency.

3. Understand the importance of input and output impedances of amplifiers.
Figure 11.1  Electronic amplifier.
Ideally, an amplifier produces an output signal with identical waveshape as the input signal, but with a larger amplitude.

\[ v_o(t) = A_v v_i(t) \]
Figure 11.2 Input waveform and corresponding output waveforms.
Inverting versus Non-inverting Amplifiers

Inverting amplifiers have negative voltage gain, and the output waveform is an inverted version of the input waveform. Non-inverting amplifiers have positive voltage gain.
Exercise 11.1

Non-inverting amplifier with a voltage gain $A_V$ of 50:

\[ v_i = 0.1 \sin(2000 \pi t) \]
\[ v_o = A_v v_i = (50)0.1 \sin(2000 \pi t) = 5 \sin(2000 \pi t) \]

Inverting amplifier with a voltage gain $A_V$ of -50:

\[ v_i = 0.1 \sin(2000 \pi t) \]
\[ v_o = A_v v_i = (-50)0.1 \sin(2000 \pi t) = -5 \sin(2000 \pi t) \]
Voltage-Amplifier Model

The input resistance $R_i$ is the equivalent resistance seen when looking into the input terminals of the amplifier. $R_o$ is the output resistance. It causes the output voltage to decrease as the load resistance becomes smaller. $A_{voc}$ is the open circuit voltage gain.
Voltage-Amplifier Model

Thevenin equivalents

Current Gain

\[ A_i = \frac{i_o}{i_i} \]

\[ A_i = \frac{i_o}{i_i} = \frac{v_o / R_L}{v_i / R_i} = A_v \frac{R_i}{R_L} \]
Power Gain

\[ G = \frac{P_o}{P_i} \]

\[ G = \frac{P_o}{P_i} = \frac{V_o I_o}{V_i I_i} = A_v A_i = (A_v)^2 \frac{R_i}{R_L} \]

*Upper case V and I indicate Root Mean Square (RMS) values*
Root Mean Square (RMS) Values

\[ V(t) = V_m \cos(\omega t + \varphi) \]

The average value of \( \cos(\omega t + \varphi) \) is zero.

This is the squared version of the signal, and its mean value is \( \frac{1}{2} \).

\[
\langle P \rangle_{\text{Average}} = \frac{1}{T} \int_0^T p \, dt = \frac{1}{T} \int_0^T \frac{V_m^2 \cos^2(\omega t + \varphi)}{R} \, dt = \frac{1}{R} \left[ \frac{1}{T} \int_0^T V_m^2 \cos^2(\omega t + \varphi) \right]
\]

\[ = \frac{V_{rms}^2}{R} \]
Root Mean Square (RMS) Values

\[ V_{rms} = \sqrt{\frac{1}{T} \int_{0}^{T} v^2(t) \, dt} \]

\[ I_{rms} = \sqrt{\frac{1}{T} \int_{0}^{T} i^2(t) \, dt} \]
Example 11.1

Find the voltage gain, the current gain and the power gain:

\[ V_i = \frac{2M\Omega}{2M\Omega + 1M\Omega} V_s = \frac{2}{3} (1mV \text{ rms}) = 0.667mV \text{ rms} \]

\[ A_{voc} V_i = 10^4 (0.667mV \text{ rms}) = 6.67V \text{ rms} \]

\[ V_o = A_{voc} V_i \frac{8\Omega}{2\Omega + 8\Omega} = \frac{4}{5} (6.67V \text{ rms}) = 5.33V \text{ rms} \]

\[ A_v = \frac{V_o}{V_i} = A_{voc} \frac{8\Omega}{2\Omega + 8\Omega} = \frac{8}{10} 10^4 = 8000 \]
Example 11.1

Note that the output signal $V_o$ is not 8000 times the input signal, $V_S$, since there is a voltage drop across the input resistance:

$$V_i = \frac{R_i}{R_i + R_s} V_S = \frac{2\, \text{M} \Omega}{2\, \text{M} \Omega + 1\, \text{M} \Omega} V_S = \frac{2}{3} V_S$$

$$\frac{V_o}{V_S} = \frac{V_o}{V_i} \frac{V_i}{V_S} = A_V \frac{V_i}{V_S} = (8000) \frac{2}{3} = 5333$$
Example 11.1

Find the current gain and the power gain:

\[ A_i = A_v \frac{R_i}{R_L} = 8000 \frac{2M\Omega}{8\Omega} = 2 \times 10^9 \]

\[ G = \text{Power Gain} = A_v A_i = (8 \times 10^3)(2 \times 10^9) = 16 \times 10^{12} \]

The current gain is very large since the input resistance (2M\(\Omega\)) allows only a small input current to flow whereas the small output resistance (8\(\Omega\)) allows a relatively large output current to flow.
Exercise 11.2

Find the voltage gain, the current gain and the power gain:

\[ V_i = \frac{2000}{2000 + 500} V_s = 0.8V_s = 16\text{mV \text{rms}} \]

\[ V_o = \frac{75}{75 + 25} A_{\text{voc}} V_i = (0.75)(500)(16\text{mV \text{rms}}) = 6V \text{ rms} \]

\[ A_v = \frac{V_o}{V_i} = (0.75)(500) = 375 \]

\[ A_i = A_v \frac{R_i}{R_L} = (375) \frac{2000}{75} = 10^4 \]

\[ G = A_v A_i = (375)(10^4) = 3.75 \times 10^6 \]
What load resistance maximizes the power gain?

From the Thevenin equivalent model we know that the maximum power delivered to a load is when the load resistance is equal to the Thevenin resistance of 25Ω.

\[
A_v = A_{voc} \frac{R_L}{R_o + R_L} = (500) \frac{25}{25 + 25} = 250
\]

\[
G = (A_v)^2 \frac{R_i}{R_L} = (250)^2 \frac{2000}{25} = 5 \times 10^6
\]
CASCADED AMPLIFIERS

Figure 11.5 Cascade connection of two amplifiers.
The overall voltage gain of cascaded amplifiers is the product of the gain of the individual stages. This is also true for the current and power gains.
Example 11.2

Find the voltage gain for each stage and for the overall cascade connection:

\[
A_{v1} = \frac{v_{o1}}{v_{i1}} = \frac{v_{i2}}{v_{i1}} = \frac{1}{v_{i1}} \left[ \frac{R_{i2}}{R_{i2} + R_{o1}} (A_{voc1} v_{i1}) \right] = \frac{R_{i2}}{R_{i2} + R_{o1}} A_{voc1} = \frac{1500}{1500 + 500} (200) = 150
\]

\[
A_{v2} = \frac{v_{o2}}{v_{i2}} = A_{voc2} \frac{R_L}{R_L + R_{o2}} = 100 \frac{100}{100 + 100} = 50
\]

\[
A_v = A_{v1} A_{v2} = (150)(50) = 7500
\]
Example 11.2

Find the current gain for each stage and for the overall cascade connection:

\[ A_{i1} = A_{v1} \frac{R_{i1}}{R_{i2}} = 150 \frac{1 \text{ M}\Omega}{1500} = 10^5 \]

\[ A_{i2} = A_{v2} \frac{R_{i2}}{R_L} = 50 \frac{1500\Omega}{100\Omega} = 750 \]

\[ A_i = A_{i1} A_{i2} = (10^5)(750) = 75 \times 10^6 \]
Find the power gain for each stage and for the overall cascade connection:

\[ G_1 = A_{v1}A_{i1} = (150)(10^5) = 1.5 \times 10^7 \]
\[ G_2 = A_{v2}A_{i2} = (50)(750) = 3.75 \times 10^4 \]
\[ G = G_1G_2 = 5.625 \times 10^{11} \]
Simplified Models for Cascaded Amplifier Stages

First, determine the voltage gain of the first stage accounting for loading by the second stage.

The overall voltage gain is the product of the gains of the separate stages.

The input impedance is that of the first stage, and the output impedance is that of the last stage.
Voltage gain of first stage accounting for loading of second stage:

\[ A_{v1} = A_{voc1} \frac{R_{i2}}{R_{i2} + R_{o1}} = 200 \frac{1500}{1500 + 500} = 150 \]

\[ A_{v2} = A_{voc} = 100 \]

\[ A_{voc} = A_{v1}A_{v2} = (150)(100) = 1.5 \times 10^4 \]
Input resistance and output resistance of the cascade:

\[ R_i = R_{i1} = 1 \text{M} \Omega \]

\[ R_o = R_{o2} = 100 \Omega \]
Simplified Model

\[ A_{voc} = 15 \times 10^3 \]
\[ R_i = 1 \text{ M\Omega} \]
\[ R_o = 100 \Omega \]
Figure 11.8 The power supply delivers power to the amplifier from several dc voltage sources.
Both power supplies are supplying power to the amplifier.

\[ P_s = V_{AA} I_A + V_{BB} I_B \]
The additional power comes from the power supply. The efficiency is given by:

$$
\eta = \frac{P_o}{P_s} \times 100
$$
Find the input power, output power, supply power and the power dissipated in the amplifier. Also find the efficiency of the amplifier.
Input power:

\[
P_i = \frac{V_i^2}{R_i} = \frac{(1 \times 10^{-3})^2}{100\, \text{k}\Omega} = 10^{-11}\, \text{W}
\]

Output power:

\[
P_B = I_B V_{BB} = (0.5\, \text{A})(15\, \text{V}) = 7.5\, \text{W}
\]

Supply power

\[
P_s = P_A + P_B = 15\, \text{W} + 7.5\, \text{W} = 22.5\, \text{W}
\]

\[
\eta = 100 \frac{P_o}{P_s} = 100 \frac{8\, \text{W}}{22.5\, \text{W}} = 35.6\%
\]