EE 135, Winter 2012

Reading: finish Chapter 1, Ulaby et.al. 6th edition

Homework #1, due Thursday, 1/19/12:

Chapter 1: problems
  1.2, 1.5, 1.9, 1.13, 1.16, 1.19, 1.26

Lecture 2
EE 135, Winter 2012

Class web site:

https://courses.soe.ucsc.edu/courses/ee135/Winter12/01

LOG IN: use Slugmail address

Discussion Session (tentative):
Wednesday, 7-8pm/ Jack’s lounge
EE 135, winter 2012

MATH DIAGNOSTIC QUIZ

to show all work.

1. Solve for $y(t)$ where $y(0) = 0$, \( \frac{dy}{dt}(0) = 1 \).
   \[ \frac{d^2y}{dt^2} + \alpha^2 y = 0, \quad \alpha = \text{constant}. \]

2. What is the complex conjugate of $z = x + jy$, where $j = \sqrt{-1}$

3. Evaluate $\int \frac{dx}{(x+a)^2} = \ ?$ where $a = \text{constant}$

4. If a scalar function $T(x, y, z) = T(\Gamma, \Theta, z)$
   where $\Gamma, \Theta$ are functions of $x, y$
   write down the expression for $\frac{dT}{dx}$.

5. Given the vectors $\vec{A}, \vec{B}$ shown below,
   evaluate the following two quantities.
   \[ \vec{A} \cdot \vec{B} = ? \]
   \[ \vec{A} \times \vec{B} = ? \]
**Static vs. Dynamic**

*Static conditions:* charges are stationary or moving, but if moving, they do so at a constant velocity.

**Table 1-3:** The three branches of electromagnetics.

<table>
<thead>
<tr>
<th>Branch</th>
<th>Condition</th>
<th>Field Quantities (Units)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Electrostatics</strong></td>
<td>Stationary charges (\partial q / \partial t = 0)</td>
<td>Electric field intensity (E) (V/m)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Electric flux density (D) (C/m²)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(D = \varepsilon E)</td>
</tr>
<tr>
<td><strong>Magnetostatics</strong></td>
<td>Steady currents (\partial I / \partial t = 0)</td>
<td>Magnetic flux density (B) (T)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Magnetic field intensity (H) (A/m)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(B = \mu H)</td>
</tr>
<tr>
<td><strong>Dynamics</strong></td>
<td>Time-varying currents (\partial I / \partial t \neq 0)</td>
<td>(E, D, B,) and (H)</td>
</tr>
<tr>
<td>(Time-varying fields)</td>
<td></td>
<td>((E, D)) coupled to ((B, H))</td>
</tr>
</tbody>
</table>

Under static conditions, electric and magnetic fields are independent, but under dynamic conditions, they become coupled.
# Material Properties

Table 1-4: Constitutive parameters of materials.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Free-space Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electrical permittivity $\varepsilon$</td>
<td>F/m</td>
<td>$\varepsilon_0 = 8.854 \times 10^{-12}$ (F/m)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\simeq \frac{1}{36\pi} \times 10^{-9}$ (F/m)</td>
</tr>
<tr>
<td>Magnetic permeability $\mu$</td>
<td>H/m</td>
<td>$\mu_0 = 4\pi \times 10^{-7}$ (H/m)</td>
</tr>
<tr>
<td>Conductivity $\sigma$</td>
<td>S/m</td>
<td>0</td>
</tr>
</tbody>
</table>
Maxwell’s Equations

\[ \nabla \cdot \mathbf{D} = \rho, \]
\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \]
\[ \nabla \cdot \mathbf{B} = 0, \]
\[ \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}. \]

Under **static** conditions, none of the quantities appearing in Maxwell’s equations are functions of time (i.e., \( \partial / \partial t = 0 \)). *This happens when all charges are permanently fixed in space, or, if they move, they do so at a steady rate so that \( \rho \) and \( \mathbf{J} \) are constant in time.* Under these circumstances, the time derivatives of \( \mathbf{B} \) and \( \mathbf{D} \) in Eqs. (4.1b) and (4.1d) vanish, and Maxwell’s equations reduce to

**Electrostatics**

\[ \nabla \cdot \mathbf{D} = \rho, \quad (4.2a) \]
\[ \nabla \times \mathbf{E} = 0. \quad (4.2b) \]

**Magnetostatics**

\[ \nabla \cdot \mathbf{B} = 0, \quad (4.3a) \]
\[ \nabla \times \mathbf{H} = \mathbf{J}. \quad (4.3b) \]

*Electric and magnetic fields become decoupled under static conditions.*

And there was light!
Traveling Waves

• Waves carry energy
• Waves have velocity
• Many waves are linear: they do not affect the passage of other waves; they can pass right through them

• Transient waves: caused by sudden disturbance
• Continuous periodic waves: repetitive source
Types of Waves

(a) Circular waves

(b) Plane and cylindrical waves

(c) Spherical wave
Parameters that Describe a Wave

- **Amplitude** \((A)\) magnitude of disturbance
- **Phase** \((\varphi)\) Point on wave of equivalent disturbance
- **Period** \((T)\) Time between successive maxima at fixed point in space
- **Frequency** \((f)\) \(f = 1/T\) number of maxima/sec at fixed point in space
- **Wavelength** \((\lambda)\) Distance between successive maxima at fixed point in time
- **Velocity** \((u)\) phase velocity of wave

\[ u = \frac{\lambda}{T} = f\lambda, \text{ independent of } A, \varphi \]
How to Represent a wave?

wave disturbance: \( h(x,t) = f(ut \pm x) \) / 1 dimension

where - is a wave moving right to left
+ is a wave moving left to right

wave moves with “phase” velocity \( u = \frac{\lambda}{T} \rightarrow \frac{dx}{dt} \)

we can write:

\[ ut \pm x = \frac{\lambda}{T} t \pm x = \left( \frac{\xi}{\lambda} \pm \frac{x}{\lambda} \right) \lambda \]

\[ \therefore f(ut \pm x) \Rightarrow f\left( \frac{\xi}{\lambda} \pm \frac{x}{\lambda} \right) \]

for a cosine wave,

\[ y(x,t) = A \cos\left( \frac{2\pi}{T} t \pm 2\pi \frac{x}{\lambda} \right) \]

2\( \pi \) is there so we go through 1 cycle
when \( \Delta t = T \) or \( \Delta x = \lambda \).
Sinusoidal Waves in Lossless Media

A medium is said to be lossless if it does not attenuate the amplitude of the wave traveling within it or on its surface.

\[ y(x, t) = A \cos \left( \frac{2\pi t}{T} - \frac{2\pi x}{\lambda} + \phi_0 \right) \text{ (m)}, \quad (1.17) \]

where \( A \) is the amplitude of the wave, \( T \) is its time period, \( \lambda \) is its spatial wavelength, and \( \phi_0 \) is a reference phase.

Note: the phase is in radians.

**Figure 1-12:** Plots of \( y(x, t) = A \cos \left( \frac{2\pi t}{T} - \frac{2\pi x}{\lambda} \right) \) as a function of (a) \( x \) at \( t = 0 \) and (b) \( t \) at \( x = 0 \).
Phase velocity

If we select a fixed height $y_0$ and follow its progress, then

$$y_0 = y(x, t) = A \cos \left( \frac{2\pi t}{T} - \frac{2\pi x}{\lambda} + \phi_0 \right)$$

where

$$\phi(x, t) = \left( \frac{2\pi t}{T} - \frac{2\pi x}{\lambda} + \phi_0 \right) \text{ (rad).}$$

Take derivative

$$\frac{2\pi}{T} - \frac{2\pi x}{\lambda} = \cos^{-1} \left( \frac{y_0}{A} \right) = \text{ constant}$$

Take time derivative

$$\frac{2\pi}{T} - \frac{2\pi}{\lambda} \frac{dx}{dt} = 0$$

$$u_p = \frac{dx}{dt} = \frac{\lambda}{T} \text{ (m/s)}$$

Figure 1-13: Plots of $y(x, t) = A \cos \left( \frac{2\pi t}{T} - \frac{2\pi x}{\lambda} \right)$ as a function of $x$ at (a) $t = 0$, (b) $t = T/4$, and (c) $t = T/2$. Note that the wave moves in the $+x$-direction with a velocity $u_p = \lambda/T$. 
Wave Frequency and Period

The frequency of a sinusoidal wave, $f$, is the reciprocal of its time period $T$:

$$f = \frac{1}{T} \quad \text{(Hz).} \quad (1.26)$$

Combining the preceding two equations yields

$$u_p = f \lambda \quad \text{(m/s).} \quad (1.27)$$

Using Eq. (1.26), Eq. (1.20) can be rewritten in a more compact form as

$$y(x, t) = A \cos \left(2\pi ft - \frac{2\pi}{\lambda} x\right) = A \cos(\omega t - \beta x), \quad (1.28)$$

where $\omega$ is the angular velocity of the wave and $\beta$ is its phase constant (or wavenumber), defined as

$$\omega = 2\pi f \quad \text{(rad/s),} \quad (1.29a)$$

$$\beta = \frac{2\pi}{\lambda} \quad \text{(rad/m).} \quad (1.29b)$$
Direction of Wave Travel

\[ y(x, t) = A \cos(\omega t - \beta x) \quad \text{Wave travelling in } +x \text{ direction} \]

\[ y(x, t) = A \cos(\omega t + \beta x) \quad \text{Wave travelling in } -x \text{ direction} \]

\textbf{+x direction:} if coefficients of } t \text{ and } x \text{ have opposite signs}

\textbf{−x direction:} if coefficients of } t \text{ and } x \text{ have same sign (both positive or both negative)}
Figure 1-14: Plots of \( y(0, t) = A \cos \left( \frac{2\pi t}{T} + \phi_0 \right) \) for three different values of the reference phase \( \phi_0 \).

\[
y(x, t) = A \cos(\omega t - \beta x + \phi_0)
\]

When its value is positive, \( \phi_0 \) signifies a phase lead in time, and when it is negative, it signifies a phase lag.
Superposition

If waves independent of one another (linear)
then:

\[ \text{Disturbance} = \sum \text{(disturbance)}_i \quad (\text{from last lecture}) \]

this gives rise to "interference" phenomena.

Where max. and min. disturbances can cancel one another.

Example of use of this?
CD Module 1.1 Sinusoidal Waveforms Learn how the shape of the waveform is related to the amplitude, frequency, and reference phase angle of a sinusoidal wave.

The waveform shown in red is a reference wave given by $y = 5 \cos(4\pi t)$:
- amplitude = 5 volts, $f = 2$ Hz, and $\phi_0 = 0$.

For comparison, you can generate and display in blue a waveform given by $y = A \cos(2\pi ft + \phi_0)$
- by specifying its attributes in the Input Panel.

Input for blue wave
- Amplitude $A = 5.0$ [volts]
- Frequency $f = 3.207$ [Hz]
- $\phi_0 = 60.84^\circ$
Wave Travel in Lossy Media

\[ y(x, t) = Ae^{-\alpha x} \cos(\omega t - \beta x + \phi_0) \]
CD Module 1.2 Traveling Waves  Learn how the shape of a traveling wave is related to its frequency and wavelength, and to the attenuation constant of the medium.

The traveling wave shown in red is given by \( y = 5\cos(\pi t - \pi x) \) volts.
Thus, its frequency is 0.5 Hz, its wavelength is 2 cm, and its reference phase angle \( \Phi_0 = 0 \), with \( t = 0 \) defined as the time the animation is started.

For comparison, you can generate and display in green a waveform given by 
\( y = 5e^{-0.5x}\cos(2\pi ft - 2\pi x/\lambda) \) volts
by specifying its attributes in the Input Panel.
**Example 1-1: Sound Wave in Water**

**Given:** sinusoidal sound wave traveling in the positive x-direction in water

Wave amplitude is 10 N/m², and \( p(x, t) \) was observed to be at its maximum value at \( t = 0 \) and \( x = 0.25 \) m. Also \( f = 1 \) kHz, \( u_p = 1.5 \) km/s.

**Determine:** \( p(x,t) \)

**Solution:**

\[
p(x, t) = A \cos\left(\frac{2\pi}{T} t - \frac{2\pi}{\lambda} x + \phi_0\right) \quad \text{(N/m²)}.
\]

The amplitude \( A = 10 \) N/m², \( T = 1/f = 10^{-3} \) s, and from \( u_p = f \lambda \).

\[
\lambda = \frac{u_p}{f} = \frac{1.5 \times 10^3}{10^3} = 1.5 \text{ m}.
\]

Hence,

\[
p(x, t) = 10 \cos\left(2\pi \times 10^3 t - \frac{4\pi}{3} x + \phi_0\right) \quad \text{(N/m²)}.
\]

Since at \( t = 0 \) and \( x = 0.25 \) m, \( p(0.25, 0) = 10 \) N/m², we have

\[
10 = 10 \cos\left(\frac{-4\pi}{3} 0.25 + \phi_0\right) = 10 \cos\left(\frac{-\pi}{3} + \phi_0\right),
\]

which yields the result \( (\phi_0 - \pi/3) = \cos^{-1}(1), \) or \( \phi_0 = \pi/3 \).

Hence,

\[
p(x, t) = 10 \cos\left(2\pi \times 10^3 t - \frac{4\pi}{3} x + \frac{\pi}{3}\right) \quad \text{(N/m²)}.
\]
A laser beam of light propagating through the atmosphere is characterized by an electric field given by

\[ E(x, t) = 150e^{-0.03x} \cos(3 \times 10^{15}t - 10^7x) \quad \text{(V/m)}, \]

where \( x \) is the distance from the source in meters. The attenuation is due to absorption by atmospheric gases. Determine:

(a) the direction of wave travel,
(b) the wave velocity, and
(c) the wave amplitude at a distance of 200 m.

**Solution:** (a) Since the coefficients of \( t \) and \( x \) in the argument of the cosine function have opposite signs, the wave must be traveling in the \(+x\)-direction.

(b) \[ u_p = \frac{\omega}{\beta} \]

\[ = \frac{3 \times 10^{15}}{10^7} \]

\[ = 3 \times 10^8 \text{ m/s}, \]

which is equal to \( c \), the velocity of light in free space.

(c) At \( x = 200 \) m, the amplitude of \( E(x, t) \) is

\[ 150e^{-0.03 \times 200} = 0.37 \quad \text{(V/m)}. \]
The EM Spectrum

\[ \lambda = \frac{c}{f} \]
Tech Brief 1: LED Lighting

**Incandescence** is the emission of light from a hot object due to its temperature.

**Fluoresce** means to emit radiation in consequence to incident radiation of a shorter wavelength.

When a voltage is applied in a forward-biased direction across an LED diode, current flows through the junction and some of the streaming electrons are captured by positive charges (holes). Associated with each electron-hole recombining act is the release of energy in the form of a photon.

Figure TF1-1: (a) Incandescent light bulb; (b) fluorescent mercury vapor lamp; (c) white LED.
Tech Brief 1: LED Basics

Figure TF1-4: Photons are emitted when electrons combine with holes.
Tech Brief 1: Light Spectra

Figure TF1-2: Spectra of common sources of visible light.
Two ways to generate a broad spectrum, but the phosphor-based approach is less expensive to fabricate because it requires only one LED instead of three.
# Tech Brief 1: LED Lighting Cost Comparison

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Incandescent</th>
<th>Fluorescent</th>
<th>White LED</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Circa 2010</td>
</tr>
<tr>
<td>Luminous Efficacy (lumens/W)</td>
<td>~12</td>
<td>~40</td>
<td>~70</td>
</tr>
<tr>
<td>Useful Lifetime (hours)</td>
<td>~1000</td>
<td>~20,000</td>
<td>~60,000</td>
</tr>
<tr>
<td>Purchase Price</td>
<td>~$1.50</td>
<td>~$5</td>
<td>~$10</td>
</tr>
<tr>
<td>Estimated Cost over 10 Years</td>
<td>~$410</td>
<td>~$110</td>
<td>~$100</td>
</tr>
</tbody>
</table>

**Figure TF1-7:** Even though the initial purchase price of a white LED is several times greater than that of the incandescent light bulb, the total 10-year cost of using the LED is only one-fourth of the incandescent’s (in 2010), and expected to decrease to one-tenth by 2025.
Energy Efficiency of Light Production

Overall efficiency for conversion of chemical energy to light energy is

\[ E_1 \times E_2 \times E_3 = 0.35 \times 0.92 \times 0.024 = 0.8\% \]

*Figure TF1-3:* Lighting efficiency. (Source: National Research Council, 2009.)
CD/DVD Player

basic principle of CD player

LED source

50% mirror

inc. ref.

detector

"land" silvered.

transparent plastic

"pit"

CD disk surface

height of "land" chosen such that when light hits "land", reflected wave is "in phase" (bright) = 1 with incident wave

when light hits "pit", reflected wave is "out of phase" (dark) = 0 with incident wave
Complex Numbers

We will find it is useful to represent sinusoids as complex numbers.

\[ z = x + jy \]  \hspace{1cm} \text{Rectangular coordinates} \hspace{1cm} \text{Re}(z) = x \\
\[ z = |z|e^{j\theta} \]  \hspace{1cm} \text{Polar coordinates} \hspace{1cm} \text{Im}(z) = y \\
\[ j = \sqrt{-1} \]

Relations based on Euler’s Identity:

\[ e^{\pm j\theta} = \cos \theta \pm j \sin \theta \]
# Relations for Complex Numbers

## Euler’s Identity:

\[ e^{j\theta} = \cos \theta + j \sin \theta \]

\[
\begin{align*}
\sin \theta &= \frac{e^{j\theta} - e^{-j\theta}}{2j} \\
\cos \theta &= \frac{e^{j\theta} + e^{-j\theta}}{2}
\end{align*}
\]

- \[ z = x + jy = |z|e^{j\theta} \]
- \[ z^* = x - jy = |z|e^{-j\theta} \]

\[
\begin{align*}
x &= \Re(z) = |z| \cos \theta \\
y &= \Im(z) = |z| \sin \theta \\
z^n &= |z|^n e^{jn\theta} \\
x &= \text{Re}(z) = |z| \cos \theta \\
y &= \text{Im}(z) = |z| \sin \theta \\
z^{1/2} &= \pm |z|^{1/2} e^{j\theta/2}
\end{align*}
\]

- \[ z_1 = x_1 + jy_1 \]
- \[ z_2 = x_2 + jy_2 \]
- \[ z_1 = z_2 \iff x_1 = x_2 \text{ and } y_1 = y_2 \]
- \[ z_1 z_2 = |z_1||z_2|e^{j(\theta_1 + \theta_2)} \]
- \[ \frac{z_1}{z_2} = \frac{|z_1|}{|z_2|} e^{j(\theta_1 - \theta_2)} \]

- \[ -1 = e^{j\pi} = e^{-j\pi} = 1 \angle \pm 180^\circ \]
- \[ j = e^{j\pi/2} = 1 \angle 90^\circ \]
- \[ -j = e^{-j\pi/2} = 1 \angle -90^\circ \]
- \[ \sqrt{j} = \pm e^{j\pi/4} = \pm \frac{1 + j}{\sqrt{2}} \]
- \[ -\sqrt{-j} = \pm e^{-j\pi/4} = \pm \frac{(1 - j)}{\sqrt{2}} \]

Learn how to perform these with your calculator/computer.
CD Module 1.4 Complex Numbers Use this module to improve your two-way rectangular/polar conversion of complex numbers.

Module 1.4 Complex Numbers

Input
Select: Rectangular to Polar

Output

\[|z| = 3.905\]
\[\theta = 140.194^\circ\]
Time Varying Circuits/Signals

\[ N_S(t) = V_0 \sin(\omega t + \varphi_0) \]
\[ R \dot{i}(t) + \frac{1}{C} \int i(t) \, dt = N_S(t) \quad \text{(time domain)} \]

solve for \( i(t) \)...

— easier with "phasors"
A *domain transformation* is a mathematical process that converts a set of variables from their domain into a corresponding set of variables defined in another domain.

1. The phasor-analysis technique transforms equations from the *time domain* to the *phasor domain*.

2. *Integro-differential* equations get converted into *linear equations* with no sinusoidal functions.

3. After solving for the desired variable--such as a particular voltage or current--in the *phasor domain*, conversion back to the *time domain* provides the same solution that would have been obtained had the original integro-differential equations been solved entirely in the *time domain*. 
Phasor Domain

\[ v(t) = V_0 \cos(\omega t + \phi) \]

\[ = \Re[ V_0 e^{j\phi} e^{j\omega t} ] \]

Phasor counterpart of \( v(t) \)

<table>
<thead>
<tr>
<th>Time Domain</th>
<th>Phasor Domain</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v(t) = V_0 \cos \omega t )</td>
<td>( V = V_0 )</td>
</tr>
<tr>
<td>( v(t) = V_0 \cos(\omega t + \phi) )</td>
<td>( V = V_0 e^{j\phi} ).</td>
</tr>
</tbody>
</table>

If \( \phi = -\pi/2 \),

\[ v(t) = V_0 \cos(\omega t - \pi/2) \quad \leftrightarrow \quad V = V_0 e^{-j\pi/2}. \]
Time and Phasor Domain

It is much easier to deal with exponentials in the phasor domain than sinusoidal relations in the time domain.

<table>
<thead>
<tr>
<th>( x(t) )</th>
<th>( X )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A \cos \omega t )</td>
<td>( A )</td>
</tr>
<tr>
<td>( A \cos(\omega t + \phi) )</td>
<td>( Ae^{j\phi} )</td>
</tr>
<tr>
<td>( -A \cos(\omega t + \phi) )</td>
<td>( Ae^{j(\phi \pm \pi)} )</td>
</tr>
<tr>
<td>( A \sin \omega t )</td>
<td>( Ae^{-j\pi/2} = -jA )</td>
</tr>
<tr>
<td>( A \sin(\omega t + \phi) )</td>
<td>( Ae^{j(\phi - \pi/2)} )</td>
</tr>
<tr>
<td>( -A \sin(\omega t + \phi) )</td>
<td>( Ae^{j(\phi + \pi/2)} )</td>
</tr>
<tr>
<td>( \frac{d}{dt} (x(t)) )</td>
<td>( j\omega X )</td>
</tr>
<tr>
<td>( \frac{d}{dt} [A \cos(\omega t + \phi)] )</td>
<td>( j\omega Ae^{j\phi} )</td>
</tr>
<tr>
<td>( \int x(t) , dt )</td>
<td>( \frac{1}{j\omega} X )</td>
</tr>
<tr>
<td>( \int A \cos(\omega t + \phi) , dt )</td>
<td>( \frac{1}{j\omega} Ae^{j\phi} )</td>
</tr>
</tbody>
</table>

Just need to track magnitude/phase, knowing that everything is at frequency \( \omega \).
PHASORS

\[ z(t) = z_0 \omega^d (\omega t + \omega) \]

\[ = \text{Re} \left[ z_0 e^{j(\omega t + \omega)} \right] \]

\[ = \text{Re} \left[ z_0 e^{j\omega t} e^{j\omega} \right] \]

\[ = \tilde{z} \text{ a phasor, can be complex} \]

\[ \tilde{z}(t) = \frac{\text{Re} \left[ \tilde{z} e^{j\omega t} \right]}{\text{phasor rep. of } z(t)} \]

Example:

\[ i(t) = \text{Re} \left[ \tilde{i} e^{j\omega t} \right] \]

\[ \frac{di}{dt} = \frac{d}{dt} \text{Re} \left[ \tilde{i} e^{j\omega t} \right] = \text{Re} \left[ \tilde{i} \cdot j\omega e^{j\omega t} \right] \]

phasor rep. of \( \frac{di}{dt} \)
Phasor Relation for Resistor

Current through resistor

Time domain
\[ i = I_m \cos(\omega t + \phi) \]
\[ v = iR = R I_m \cos(\omega t + \phi) \]

Phasor Domain

\[ V = RI \]
\[ = R I_m \angle \phi \]
Phasor Relation for Inductors

Time Domain
\[ v = L \frac{di}{dt} \]
\[ V = j\omega LI \]

Frequency Domain
\[ i \quad \rightarrow \quad I \]
\[ + \quad \rightarrow \quad + \]
\[ V \quad \rightarrow \quad V \]
\[ + \quad \rightarrow \quad + \]
\[ L \quad \rightarrow \quad L \]

Time domain
\[ v = L \frac{di}{dt} \]

Phasor Domain
\[ v_L = \Re[V_L e^{j\omega t}] \]
\[ i_L = \Re[I_L e^{j\omega t}] \]

Consequently,
\[ \Re[V_L e^{j\omega t}] = L \frac{d}{dt} \left[ \Re(I_L e^{j\omega t}) \right] \]
\[ = \Re[j\omega LI_L e^{j\omega t}] \]

which leads to
\[ V_L = j\omega LI_L \]

and
\[ Z_L = \frac{V_L}{I_L} = j\omega L. \]
Phasor Relation for Capacitors

**Time Domain**

\[ i = C \frac{dv}{dt} \]

**Frequency Domain**

\[ I = j\omega CV \]

**Time domain**

\[ i = C \frac{dv}{dt} \]

**Phasor Domain**

\[ I_C = j\omega CV_C \]

\[ Z_C = \frac{V_C}{I_C} = \frac{1}{j\omega C} \]