EE 135, Winter 2012

Reading: finish Chapter 1, Ulaby et.al. 6th edition
start Chapter 2, sections 2.1-2.4.pp 48-60.

Homework #1, due Thursday, 1/19/12:

Chapter 1: problems
1.2, 1.5, 1.9, 1.13, 1.16, 1.19, 1.26

Lecture 3
EE 135, Winter 2012

Class web site:

https://courses.soe.ucsc.edu/courses/ee135/Winter12/01

LOG IN: use Slugmail address and password

Discussion Session:
Wednesday, 7-8pm/ Jack’s lounge
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<th>Monday 11am-2pm</th>
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<tr>
<td>Eric</td>
<td>Yu</td>
<td>x</td>
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</table>
EE 135, Winter 2012
Math Diagnostic Quiz
Solutions 1

1. solve for \( y(t) \) if \( y(0) = 0 \), \( y'(0) = 1 \)

\[ \frac{d^2 y}{dt^2} + a^2 y = 0 \quad \text{where } a = \text{constant} \]

solution of form \( y(t) = A \sin(at) + B \cos bt \)

\[ y(0) = B = 0 \]

\[ y'(0) = aA \sin at + (-bB) \cos bt \]

\[ y'(0) = aA = 1 \quad \Rightarrow \quad A = \frac{1}{a} \]

\[ y(t) = \frac{1}{a} \sin(at) \]

2. Complex conjugates of \( z = x + jy \) where \( j = \sqrt{-1} \)

\[ z^* = x - jy \]

3. \[ \int \frac{dx}{(x+a)^2} = ? \quad \text{where } a = \text{constant} \]

\[ \int \frac{dx}{x^n} = - \frac{1}{n-1} \frac{1}{x^{n-1}} \]

\[ \left[ \frac{dx}{(x+a)^n} = - \frac{1}{n-1} \frac{1}{(x+a)^{n-1}} \right] \]
EE 135. Winter 2012
Math Diagnostic Quiz
Solutions 2.

4. If scalar function $T(x, y, z) = T(r, \theta, z)$
   where $r, \theta, z$ are functions of $x, y, z$,
   
   \[
   \frac{\partial T(x, y, z)}{\partial x} = \frac{\partial T}{\partial r} \frac{\partial r}{\partial x} + \frac{\partial T}{\partial \theta} \frac{\partial \theta}{\partial x} + \frac{\partial T}{\partial z} \frac{\partial z}{\partial x}
   \]

   If $x, y, z$ are rect. coordinates
   and $r, \theta, z$ are cyl. coordinates then $\frac{\partial z}{\partial x} = 0$.

5. If $\vec{A}, \vec{B}$ are as shown below (\vec{A}, \vec{B} vectors)

   \[\vec{A} - \vec{B} = AB \cos \theta\]
   a scalar quantity, the “dot” product

   \[\vec{A} \times \vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}\]
   in rectangular coordinates

   \[\vec{A} \times \vec{B} = \hat{x}(A_y B_z - A_z B_y) + \hat{y}(A_z B_x - A_x B_z) + \hat{z}(A_x B_y - A_y B_x)\]
   a vector directed into the plane of the page

   \[|\vec{A} \times \vec{B}| = AB \sin \theta\]
Time Varying Circuits/Signals

\[ N_s(t) = V_0 \sin(\omega t + \phi_0) \]

\[ R \dot{i}(t) + \frac{1}{C} \int i(t) \, dt = N_s(t) \quad \text{(time domain)} \]

Solve for \( i(t) \)...

--- easier with "phasors"
Phasor Domain

A domain transformation is a mathematical process that converts a set of variables from their domain into a corresponding set of variables defined in another domain.

1. The phasor-analysis technique transforms equations from the time domain to the phasor domain.

2. Integro-differential equations get converted into linear equations with no sinusoidal functions.

3. After solving for the desired variable--such as a particular voltage or current--in the phasor domain, conversion back to the time domain provides the same solution that would have been obtained had the original integro-differential equations been solved entirely in the time domain.
Phasor Domain

\[ v(t) = V_0 \cos(\omega t + \phi) \]
\[ = \Re\left[ V_0 e^{j\phi} e^{j\omega t} \right] \]

Phasor counterpart of \( v(t) \)

<table>
<thead>
<tr>
<th>Time Domain</th>
<th>Phasor Domain</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v(t) = V_0 \cos \omega t )</td>
<td>( V = V_0 )</td>
</tr>
<tr>
<td>( v(t) = V_0 \cos(\omega t + \phi) )</td>
<td>( V = V_0 e^{j\phi} ).</td>
</tr>
</tbody>
</table>

If \( \phi = -\pi/2 \),

\[ v(t) = V_0 \cos(\omega t - \pi/2) \quad \leftrightarrow \quad V = V_0 e^{-j\pi/2}. \]
It is much easier to deal with exponentials in the phasor domain than sinusoidal relations in the time domain.

Just need to track magnitude/phase, knowing that everything is at frequency $\omega$.

<table>
<thead>
<tr>
<th>$x(t)$</th>
<th>$X$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A \cos \omega t$</td>
<td>$A$</td>
</tr>
<tr>
<td>$A \cos(\omega t + \phi)$</td>
<td>$Ae^{j\phi}$</td>
</tr>
<tr>
<td>$-A \cos(\omega t + \phi)$</td>
<td>$Ae^{j(\phi \pm \pi)}$</td>
</tr>
<tr>
<td>$A \sin \omega t$</td>
<td>$Ae^{-j\pi/2} = -jA$</td>
</tr>
<tr>
<td>$A \sin(\omega t + \phi)$</td>
<td>$Ae^{j(\phi - \pi/2)}$</td>
</tr>
<tr>
<td>$-A \sin(\omega t + \phi)$</td>
<td>$Ae^{j(\phi + \pi/2)}$</td>
</tr>
<tr>
<td>$\frac{d}{dt}(x(t))$</td>
<td>$j\omega X$</td>
</tr>
<tr>
<td>$\frac{d}{dt}[A \cos(\omega t + \phi)]$</td>
<td>$j\omega Ae^{j\phi}$</td>
</tr>
<tr>
<td>$\int x(t) , dt$</td>
<td>$\frac{1}{j\omega} X$</td>
</tr>
<tr>
<td>$\int A \cos(\omega t + \phi) , dt$</td>
<td>$\frac{1}{j\omega} Ae^{j\phi}$</td>
</tr>
</tbody>
</table>
PHASORS

\[ z(t) = z_0 e^{j(w t + \phi)} \]
\[ z(t) = \text{Re} \left[ z_0 e^{j(w t + \phi)} \right] \]
\[ = \text{Re} \left[ z_0 e^{j\phi} e^{jw t} \right] \]
\[ = \frac{z_0}{e^{j\phi}} \text{ a phasor (can be complex but time-independent)} \]
\[ = z(t) = \text{Re} \left[ z_0 e^{jw t} \right] \]

Examples:

diff': \[ i(t) = \text{Re} \left[ i_0 e^{jw t} \right] \]
\[ \frac{di}{dt} = \frac{d}{dt} \text{Re} \left[ i_0 e^{jw t} \right] = \text{Re} \left[ \frac{d}{dt} \left( i_0 e^{jw t} \right) \right] \]
\[ = \text{Re} \left[ i_0 jw e^{jw t} \right] \]
\[ \text{phasor of } \frac{di}{dt} \]

integ': \[ \int i(t) dt = \int \text{Re} \left[ i_0 e^{jw t} \right] dt = \text{Re} \left[ \int i_0 e^{jw t} dt \right] \]
\[ = \text{Re} \left[ i_0 \frac{1}{jw} e^{jw t} \right] \]
\[ = \text{phasor of } \int i dt \]
Phasor Relation for Resistors

Current through resistor

**Time domain**

\[ i = I_m \cos(\omega t + \phi) \]
\[ \nu = iR = R I_m \cos(\omega t + \phi) \]

**Phasor Domain**

\[ \mathbf{V} = R \mathbf{I} \]
\[ = R I_m \angle \phi \]
Phasor Relation for Inductors

**Time Domain**

\[ v = L \frac{di}{dt} \]

**Frequency Domain**

\[ V = j\omega LI \]

**Phasor Domain**

\[ v_L = \Re[V_L e^{j\omega t}] \]

and

\[ i_L = \Re[I_L e^{j\omega t}] \]

Consequently,

\[ \Re[V_L e^{j\omega t}] = L \frac{d}{dt} \Re[I_L e^{j\omega t}] \]

\[ = \Re[j\omega LI_L e^{j\omega t}] \]

which leads to

\[ V_L = j\omega LI_L \]

and

\[ Z_L = \frac{V_L}{I_L} = j\omega L. \]
Phasor Relation for Capacitors

Time Domain
\[ i = C \frac{dv}{dt} \]

Frequency Domain
\[ I = j\omega CV \]

Time domain
\[ i = C \frac{dv}{dt} \]

Phasor Domain
\[ I_C = j\omega CV_C \]

\[ Z_C = \frac{V_C}{I_C} = \frac{1}{j\omega C} \]
ac Phasor Analysis: General Procedure

**Step 1**
Adopt Cosine Reference (Time Domain)

\[ v_s(t) = 12 \sin(\omega t - 45^\circ) \] (V)

**Step 2**
Transfer to Phasor Domain

\[ i \rightarrow I \]
\[ v \rightarrow V \]
\[ R \rightarrow Z_R = R \]
\[ L \rightarrow Z_L = j\omega L \]
\[ C \rightarrow Z_C = 1/j\omega C \]

\[ V_s = 12e^{-j135^\circ} \] (V)

**Step 3**
Cast Equations in Phasor Form

\[ I \left( R + \frac{1}{j\omega C} \right) = V_s \]

**Step 4**
Solve for Unknown Variable (Phasor Domain)

\[ I = \frac{V_s}{R + \frac{1}{j\omega C}} \]

**Step 5**
Transform Solution Back to Time Domain

\[ i(t) = \Re\{Ie^{j\omega t}\} = 6 \cos(\omega t - 105^\circ) \] (mA)
Example 1-4: RL Circuit

The voltage source of the circuit shown in Fig. 7-8(a) is given by

\[ v_s(t) = 15 \sin(4 \times 10^4 t - 30^\circ) \text{ V}. \]

Also, \( R = 3 \) \( \Omega \) and \( L = 0.1 \) mH. Obtain an expression for the voltage across the inductor.

**Solution:**

**Step 1:** Convert \( v_s(t) \) to the cosine reference

\[ v_s(t) = 15 \sin(4 \times 10^4 t - 30^\circ) \]
\[ = 15 \cos(4 \times 10^4 t - 30^\circ - 90^\circ) \]
\[ = 15 \cos(4 \times 10^4 t - 120^\circ) \text{ V}, \]

and its corresponding phasor \( V_s \) is given by

\[ V_s = 15e^{-j120^\circ} \text{ V}. \]

**Step 2:** Transform circuit to the phasor domain
Example 1-4: RL Circuit cont.

Step 3: Cast KVL in phasor domain

\[ R I + j \omega LI = V_s. \]

Step 4: Solve for unknown variable

\[
I = \frac{V_s}{R + j \omega L} = \frac{15e^{-j120^\circ}}{3 + j4 \times 10^4 \times 10^{-4}} = \frac{15e^{-j120^\circ}}{3 + j4} = \frac{15e^{-j120^\circ}}{5e^{j53.1^\circ}} = 3e^{-j173.1^\circ} \text{ A.}
\]

The phasor voltage across the inductor is related to \( I \) by

\[ V_L = j \omega LI \]

\[
= j 4 \times 10^4 \times 10^{-4} \times 3e^{-j173.1^\circ}
= j 12e^{-j173.1^\circ}
= 12e^{-j173.1^\circ} \cdot e^{j90^\circ} = 12e^{-j83.1^\circ} \text{ V},
\]

where we replaced \( j \) with \( e^{j90^\circ} \).

Step 5: Transform solution to the time domain

The corresponding time-domain voltage is

\[ v_L(t) = \Re[V_L e^{j\omega t}] \]

\[ = \Re[12e^{-j83.1^\circ} e^{j4 \times 10^4 t}] \]

\[ = 12 \cos(4 \times 10^4 t - 83.1^\circ) \text{ V.} \]
Table 1-5: Time-domain sinusoidal functions $z(t)$ and their cosine-reference phasor-domain counterparts $\tilde{Z}$, where $z(t) = \Re \left[ \tilde{Z} e^{j\omega t} \right]$.

<table>
<thead>
<tr>
<th>$z(t)$</th>
<th>$\tilde{Z}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A \cos \omega t$</td>
<td>$A$</td>
</tr>
<tr>
<td>$A \cos(\omega t + \phi_0)$</td>
<td>$A e^{j\phi_0}$</td>
</tr>
<tr>
<td>$A \cos(\omega t + \beta x + \phi_0)$</td>
<td>$A e^{j(\beta x + \phi_0)}$</td>
</tr>
<tr>
<td>$A e^{-\alpha x} \cos(\omega t + \beta x + \phi_0)$</td>
<td>$A e^{-\alpha x} e^{j(\beta x + \phi_0)}$</td>
</tr>
<tr>
<td>$A \sin \omega t$</td>
<td>$A e^{-j\pi/2}$</td>
</tr>
<tr>
<td>$A \sin(\omega t + \phi_0)$</td>
<td>$A e^{j(\phi_0 - \pi/2)}$</td>
</tr>
</tbody>
</table>

\[
\frac{d}{dt}(z(t)) \quad \leftrightarrow \quad j \omega \tilde{Z}
\]

\[
\frac{d}{dt}[A \cos(\omega t + \phi_0)] \quad \leftrightarrow \quad j \omega A e^{j\phi_0}
\]

\[
\int z(t) \, dt \quad \leftrightarrow \quad \frac{1}{j\omega} \tilde{Z}
\]

\[
\int A \sin(\omega t + \phi_0) \, dt \quad \leftrightarrow \quad \frac{1}{j\omega} A e^{j(\phi_0 - \pi/2)}
\]
1-7.2 Traveling Waves in the Phasor Domain

According to Table 1-5, if we set $\phi_0 = 0$, its third entry becomes

\[ A \cos(\omega t + \beta x) \leftrightarrow Ae^{j\beta x}. \quad (1.74) \]

From the discussion associated with Eq. (1.31), we concluded that $A \cos(\omega t + \beta x)$ describes a wave traveling in the negative $x$-direction.

In the phasor domain, a wave of amplitude $A$ traveling in the positive $x$-direction in a lossless medium with phase constant $\beta$ is given by the negative exponential $Ae^{-j\beta x}$, and conversely, a wave traveling in the negative $x$-direction is given by $Ae^{j\beta x}$. Thus, the sign of $x$ in the exponential is opposite to the direction of travel.
Chapter 1 Relationships

Electric field due to charge $q$ in free space

$$E = \hat{\mathbf{R}} \frac{q}{4\pi \varepsilon_0 R^2}$$

Magnetic field due to current $I$ in free space

$$B = \hat{\mathbf{\phi}} \frac{\mu_0 I}{2\pi r}$$

Plane wave

$$y(x, t) = Ae^{-\alpha x} \cos(\omega t - \beta x + \phi_0)$$

- $\alpha = 0$ in lossless medium
- Phase velocity $u_p = f\lambda = \frac{\omega}{\beta}$
- $\omega = 2\pi f$; $\beta = 2\pi / \lambda$
- $\phi_0$ = phase reference

Complex numbers

- Euler’s identity
  $$e^{j\theta} = \cos \theta + j \sin \theta$$
- Rectangular-polar relations
  $$x = |z| \cos \theta, \quad y = |z| \sin \theta, \quad |z| = \sqrt{x^2 + y^2}, \quad \theta = \tan^{-1}(y/x)$$

Phasor-domain equivalents

Table 1-5
2. TRANSMISSION LINES

Applied EM by Ulaby, Michielssen and Ravaioli
# Chapter 2 Overview

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## Objectives

Upon learning the material presented in this chapter, you should be able to:

1. Calculate the line parameters, characteristic impedance, and propagation constant of coaxial, two-wire, parallel-plate, and microstrip transmission lines.
2. Determine the reflection coefficient at the load-end of the transmission line, the standing wave pattern, and the locations of voltage and current maxima and minima.
3. Calculate the amount of power transferred from the generator to the load through the transmission line.
4. Use the Smith chart to perform transmission-line calculations.
Transmission Lines

A transmission line connects a generator to a load

Transmission lines include:
• Two parallel wires
• Coaxial cable
• Microstrip line
• Optical fiber
• Waveguide
• etc.
Is the pair of wires connecting the voltage source to the RC load a transmission line? Yes.
The wires were ignored in circuits courses. Can we always ignore them? Not always.

\[ V_{AA'} = V_g(t) = V_0 \cos \omega t \quad \text{(V)} \]

\[ V_{BB'}(t) = V_{AA'}(t - l/c) \quad \text{Delayed by } l/c \]
\[ = V_0 \cos[\omega(t - l/c)] \]
\[ = V_0 \cos(\omega t - \phi_0), \]
Transmission Line Effects

Is the pair of wires connecting the voltage source to the RC load a transmission line? Yes.
The wires were ignored in circuits courses. Can we always ignore them? Not always.

\[ V_{AA'} = V_g(t) = V_0 \cos \omega t \quad (V) \]

Signal delayed by \( l/c \)

At \( t = 0 \), and for \( f = 1 \) kHz, if:

1. \( l = 5 \) cm:
   \[ V_{BB'} = V_0 \cos(2\pi f l/c) = 0.999999999998 V_0 \]

2. But if \( l = 20 \) km:
   \[ V_{BB'} = 0.91 V_0 \]

When \( l/\lambda \) is very small, transmission-line effects may be ignored, but when \( l/\lambda \gtrsim 0.01 \), it may be necessary to account not only for the phase shift due to the time delay, but also for the presence of reflected signals that may have been bounced back by the load toward the generator.
Figure 2-3: A dispersionless line does not distort signals passing through it regardless of its length, whereas a dispersive line distorts the shape of the input pulses because the different frequency components propagate at different velocities. The degree of distortion is proportional to the length of the dispersive line.
Types of Transmission Modes

**TEM (Transverse Electromagnetic):** Electric and magnetic fields are orthogonal to one another, and both are orthogonal to direction of propagation.
Example of TEM Mode

Electric Field $E$ is radial
Magnetic Field $H$ is azimuthal
Propagation is into the page
Transmission Line Model

- \( R' \): The combined resistance of both conductors per unit length, in \( \Omega/\text{m} \),
- \( G' \): The conductance of the insulation medium between the two conductors per unit length, in \( S/\text{m} \), and
- \( L' \): The combined inductance of both conductors per unit length, in \( \text{H/\text{m}} \),
- \( C' \): The capacitance of the two conductors per unit length, in \( \text{F/m} \).
Table 2-1: Transmission-line parameters $R'$, $L'$, $G'$, and $C'$ for three types of lines.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Coaxial</th>
<th>Two-Wire</th>
<th>Parallel-Plate</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R'$</td>
<td>$\frac{R_s}{2\pi} \left( \frac{1}{a} + \frac{1}{b} \right)$</td>
<td>$\frac{2R_s}{\pi d}$</td>
<td>$\frac{2R_s}{w}$</td>
<td>$\Omega/m$</td>
</tr>
<tr>
<td>$L'$</td>
<td>$\frac{\mu}{2\pi} \ln(b/a)$</td>
<td>$\frac{\mu}{\pi} \ln \left[ (D/d) + \sqrt{(D/d)^2 - 1} \right]$</td>
<td>$\frac{\mu h}{w}$</td>
<td>H/m</td>
</tr>
<tr>
<td>$G'$</td>
<td>$\frac{2\pi \sigma}{\ln(b/a)}$</td>
<td>$\frac{\pi \sigma}{\ln \left[ (D/d) + \sqrt{(D/d)^2 - 1} \right]}$</td>
<td>$\frac{\sigma w}{h}$</td>
<td>S/m</td>
</tr>
<tr>
<td>$C'$</td>
<td>$\frac{2\pi \varepsilon}{\ln(b/a)}$</td>
<td>$\frac{\pi \varepsilon}{\ln \left[ (D/d) + \sqrt{(D/d)^2 - 1} \right]}$</td>
<td>$\frac{\varepsilon w}{h}$</td>
<td>F/m</td>
</tr>
</tbody>
</table>

Notes: (1) Refer to Fig. 2-4 for definitions of dimensions. (2) $\mu$, $\varepsilon$, and $\sigma$ pertain to the insulating material between the conductors. (3) $R_s = \sqrt{\frac{\mu}{\pi \varepsilon_c}}$. (4) $\mu_c$ and $\sigma_c$ pertain to the conductors. (5) If $(D/d)^2 \gg 1$, then $\ln \left[ (D/d) + \sqrt{(D/d)^2 - 1} \right] \simeq \ln(2D/d)$.

The pertinent constitutive parameters apply to all three lines and consist of two groups: (1) $\mu_c$ and $\sigma_c$ are the magnetic permeability and electrical conductivity of the conductors, and (2) $\varepsilon$, $\mu$, and $\sigma$ are the electrical permittivity, magnetic permeability, and electrical conductivity of the insulation material separating them.
eventually you will be able to work out all the

gains in the stuff geometries but what you will see
is this: \[ L = \frac{1}{2\pi} \ln(b/a) \]

\( \mu = \) permeability of shell

\( r = \) separating the bundle

\( C = \frac{1}{2\pi} \ln(b/a) \)

\( \varepsilon = \) permittivity of shell

\( r = \) separating the bundle

\( L' = \frac{1}{2\pi} \ln(b/a) \times \frac{2\pi}{\ln(b/a)} = \boxed{\mu \varepsilon = L' C'} \)

This holds for all geometries

note that \( \mu \varepsilon \) before the phase vel (wave vel)

of the liquid is \( u_p = \sqrt{\frac{\mu \varepsilon}{\varepsilon}} \)

Maxwell

\[ L' C' = \frac{1}{u_p^2} \]

so

Similarly we will find that

\[ \frac{G'}{C'} = \frac{\varepsilon - \mu}{\varepsilon} \]

depends on material

if insulator is air or a vacuum

\( \varepsilon = \varepsilon_o, \mu = 0, \varepsilon = 0 \rightarrow G' = 0 \)

which makes sense -

the leaves the insulator, the lines

"leakage" from 1 line to the other
Kirchhoff’s Laws

- No smoking
- $\sum u = 0$ (loop)
- No swearing
- $\sum i = 0$ (node)
- No skateboards
- No headsets
- No gum
- No talking
- No food
- No drinks
Transmission-Line Equations

\[ i(z, t) - G' \Delta z \, v(z + \Delta z, t) \]
\[ - C' \Delta z \frac{\partial v(z + \Delta z, t)}{\partial t} - i(z + \Delta z, t) = 0. \] (2.15)

Upon dividing all terms by \( \Delta z \) and taking the limit \( \Delta z \to 0 \),
Eq. (2.15) becomes a second-order differential equation:

\[ -\frac{\partial i(z, t)}{\partial z} = G' \, v(z, t) + C' \frac{\partial v(z, t)}{\partial t}. \] (2.16)

Upon dividing all terms by \( \Delta z \) and rearranging them, we obtain
\[ \left[ \frac{v(z + \Delta z, t) - v(z, t)}{\Delta z} \right] = R' i(z, t) + L' \frac{\partial i(z, t)}{\partial t}. \] (2.13)

In the limit as \( \Delta z \to 0 \), Eq. (2.13) becomes a differential equation:

\[ -\frac{\partial v(z, t)}{\partial z} = R' \, i(z, t) + L' \frac{\partial i(z, t)}{\partial t}. \] (2.14)

**ac signals: use phasors**

\[ v(z, t) = \Re \{ \tilde{V}(z) e^{j\omega t} \}, \]
\[ i(z, t) = \Re \{ \tilde{I}(z) e^{j\omega t} \}, \]

**Telegrapher's equations**

\[ -\frac{d\tilde{V}(z)}{dz} = (R' + j\omega L') \tilde{I}(z), \]
\[ -\frac{d\tilde{I}(z)}{dz} = (G' + j\omega C') \tilde{V}(z). \]
at how else signals propagate thru transmission lines.

Schematically we look at a differential element of the "lumped circuit" model.

In simple terms, the transmission line connects the source and load.

- **Source**
- **Transmission Line**
- **Load**

Look at 1 element of the line.

For the **Nth** element, the voltage is:

\[ V(z,t) \]

- **Source**
- **Load**

In this differential element:

- Input voltage: \( V(z,t) \)
- Output voltage: \( V(z+\Delta z,t) \)

We use Kirchhoff's law and circuit in voltage and current.
Then, with voltage drop from node $N \rightarrow NH$:

$$V_2(t) - V_{2i} - V_{Li} - V(z_{i+2},t) = 0 \quad \text{at any time}$$

for $N \rightarrow NH \rightarrow N$.

$$V(z_{i+2},t) = V(z_{i+2},t)$$

A little algebra gives:

$$V(z_{i+1},t) - V(z_{i+2},t) = \left[ V(z_{i+1},t) + \frac{dV(z_{i+1},t)}{dt} \right] \Delta z$$

Taking limit as $\Delta z \to 0$ gives:

Voltage eqn. 1)

$$\frac{-\Delta V(z_{i+1},t)}{\Delta z} = \left[ \frac{dV(z_{i+1},t)}{dt} \right] \frac{1}{\Delta z} \text{ as deriv with fixed gap}$$

How do the current law amid circuit:

$$I(z_{i+1},t) - C \frac{dV(z_{i+2},t)}{dt} - C' \frac{dV(z_{i+3},t)}{dt}$$

$$\text{at } NH \rightarrow I(z_{i+2},t)$$

$$\frac{\Delta I(z_{i+1},t)}{\Delta z} = \left[ \frac{dV(z_{i+1},t)}{dt} \right] \frac{1}{\Delta z}$$

Let limit $\Delta z \to 0$ then

Current eqn. 2)

$$\frac{-\Delta I(z_{i+1},t)}{\Delta z} = \left[ \frac{dV(z_{i+1},t)}{dt} \right] \frac{1}{\Delta z}$$
Eqn's (1) & (2) are called the telegraph eqn -
the trans line behavior in "time domain"

Since we can express everything in terms of time domain,
we can transform eqns into frequency eqns. (freq domain)

From before: \( V(t) = V_0 \cos(\omega t + \phi_0 - \pi / 2) \)

\[ = \text{Re} \left[ V(t) \cdot e^{j\omega t} \right] \]
\( \quad \text{voltag phasor} \quad j(\phi_0 - \pi / 2) \)
where \( V(t) = V_0 e^{j\phi} \)

Approach: write down frequency eqns, solve
then "back-transform" to the time domain.

\text{Note:} we are going thus this to see how a transmission
line functions as a funct of frequency

So write \( \tilde{V}(t) = V_0 e^{j(\phi_0 - \pi / 2)} \)

\[ v(t) = \text{Re} \left[ \tilde{V}(t) \cdot e^{j\omega t} \right] = V_0 \cos(\omega t + \phi_0 - \pi / 2) \]
\[ \vdash(t) = \text{Im} \left[ \tilde{V}(t) \cdot e^{j\omega t} \right] \]

and remembering that: \( \frac{d}{dt} \text{ in time domain} \rightarrow j\omega \text{ in phasor} \)

Then we can write down the 2 time domain diff eqns (for \( v, \vdash \)) and transform to 2 freq domain diff eqns.
\[ -\frac{\Delta V}{\Delta z} = R' \frac{\Delta A}{\Delta t} + L' \frac{\Delta I}{\Delta t} \]

\[ -\frac{\Delta I}{\Delta z} = C' \Delta V + C' \frac{\Delta V}{\Delta t} \]

Which becomes

\[
\begin{align*}
\text{frequency domain} & \quad -\frac{\Delta V}{\Delta z} = (R' + jwL') \frac{\Delta A}{\Delta z} \\
\text{phasedomain} & \quad -\frac{\Delta i}{\Delta z} = (R' + jwC') \frac{\Delta A}{\Delta z}
\end{align*}
\]

So we can combined these 1\text{\textsuperscript{st}} order eqns to give a single 2\text{\textsuperscript{nd}} order diff eqn.

\[ -\frac{\Delta^2 V}{\Delta z^2} = (R' + jwL') \frac{\Delta^2 A}{\Delta z^2} \]

Substitute this into eqn (2) above:

\[ -\frac{\Delta^2 V}{\Delta z^2} = (R' + jwL') \left[ - (C' + jwC') \right] \frac{\Delta^2 A}{\Delta z^2} \]

\[ \frac{\Delta^2 V}{\Delta z^2} - (R' + jwL')(C' + jwC') \frac{\Delta A}{\Delta z} = 0 \]

If we call \( \Delta^2 = (R' + jwL')(C' + jwC') \) then

\[ \frac{\Delta^2 V}{\Delta z^2} - \Delta^2 \frac{\Delta A}{\Delta z} = 0 \]

Simple 2\text{\textsuperscript{nd}} order diff eqn.
Now do same by differentiating (2) w.r.t. ξ we get

\[-\frac{\partial^2 \tilde{V}}{\partial \xi^2} = (6' + j\omega c_l) \frac{\partial \tilde{V}}{\partial \xi}\]

Substituting (1) into (2)

\[-\frac{\partial^2 \tilde{V}}{\partial \xi^2} = (6' + j\omega c_l) \left[-\frac{\partial^2 \tilde{V}}{\partial \xi^2} + \frac{\partial \tilde{V}}{\partial \xi}\right]\]

\[-\frac{\partial \tilde{V}}{\partial \xi} - \frac{\partial^2 \tilde{V}}{\partial \xi^2} = 0\] with ξ² same as before

These are the two “wave eqns” for the current & voltage

Phase - \(\gamma = \) complex propagation constant:

\(\gamma = \alpha + j\beta\)

\(\Rightarrow \alpha = \) attenuation coefficient (const) real

of the line = \(Re(\gamma)\)

in units of \(\frac{1}{\text{metres}}\)

\(\beta = \) phase constant of line (in radians/m)

Note: text has units of \(\beta\) as

Note: we can write \(\alpha, \beta\) now

in terms of lumped (infinite)

circuit elements

\(\gamma = \sqrt{(R' + j\omega L)(6' + j\omega C_l)}\)

\(\alpha = \Re\left[\sqrt{\frac{R'}{R' + j\omega L}}\right]\), \(\beta = \Im\left[\sqrt{\frac{R'}{R' + j\omega L}}\right]\)

Note: \(\alpha, \beta\) depend upon the equivalent circuit elements, i.e., the properties of the trans. line.
so we have eqns for the voltage and current phasors
\[
\begin{align*}
\frac{\partial^2 \tilde{V}}{\partial z^2} - \alpha^2 \tilde{V} &= 0 \\
\frac{\partial^2 \tilde{I}}{\partial z^2} - \alpha^2 \tilde{I} &= 0
\end{align*}
\]
where \( \alpha \) can be complex

the solution is of the form:
\[
\tilde{V}(z) = V_0^+ e^{-\alpha z} + V_0^- e^{\alpha z}
\]
\[
\tilde{I}(z) = I_0^+ e^{-\alpha z} + I_0^- e^{\alpha z}
\]

we can interpret \( \alpha \) as \( \alpha = \omega + j \beta \) since \( e^{-j \beta z} \) gets attenuated.

\( \omega \) is the \( +z \) wave, \( \beta \) is the \( -z \) wave.

\( V_0^+, I_0^+ \) are the \( +z \) propagating waves.
\( V_0^-, I_0^- \) are the \( -z \) waves.

we can show that these solutions are ok.
\[
\begin{align*}
\frac{\partial^2 \tilde{V}}{\partial z^2} &= V_0^+ (\sigma^2) e^{-\alpha z} + V_0^- (\sigma^2) e^{\alpha z} \\
\frac{\partial^2 \tilde{I}}{\partial z^2} &= I_0^+ (\sigma^2) e^{-\alpha z} + I_0^- (\sigma^2) e^{\alpha z} = \alpha^2 [V_0^+ e^{-\alpha z} + V_0^- e^{\alpha z}]
\end{align*}
\]

\( \alpha^2 \tilde{V} \) is real.

now relate the current phasor to voltage phasors

\[
\begin{align*}
-\frac{\partial \tilde{V}}{\partial z} &= (R'L + j \omega L') \tilde{I} \\
-\frac{\partial \tilde{I}}{\partial z} &= (\sigma' + j \omega C') \tilde{V}
\end{align*}
\]
Get un solved:

\[ I = -\frac{3V}{\Delta} \frac{1}{R + j\omega L} \]

\[ = \frac{\Delta}{-\Delta V_0 e^{-\frac{\Delta^2}{2}} + \Delta V_0 e^{\frac{\Delta^2}{2}}} \]

\[ = \frac{\Delta}{R + j\omega L} \left[ V_0 e^{-\frac{\Delta^2}{2}} - V_0 e^{\frac{\Delta^2}{2}} \right] \]

But \( \Delta = \sqrt{(R + j\omega L)(g + j\omega L)} \)

\[ = \frac{\Delta}{R + j\omega L} \frac{(R + j\omega L)(g + j\omega L)}{(R + j\omega L)^2} = \frac{g + j\omega L}{R + j\omega L} \]

---

If we define the characteristic impedance of the line to be

\[ Z_0 = \sqrt{\frac{R + j\omega L}{g + j\omega L}} \]

Then \( I = \frac{1}{Z_0} \left[ V_0 e^{-\frac{\Delta^2}{2}} - V_0 e^{\frac{\Delta^2}{2}} \right] \)

\[ I = I_0 e^{-\frac{\Delta^2}{2}} - I_0 e^{\frac{\Delta^2}{2}} \quad \text{where} \quad I_0 = \frac{V_0}{Z_0} \]

\[ I_0^- = \frac{V_0}{Z_0} \]

---

Note: the characteristic impedance

\[ Z_0 = \frac{V_0^+}{I_0^+} \]

\[ Z_0 = \frac{V_0^-}{I_0^-} \]

\[ I_0^- = \frac{V_0}{Z_0} \]

\[ \neq \frac{V}{I} \text{ unless the wave } \neq 0 \]
the current phasor is:

\[
\tilde{I}(t) = \frac{V_0^+ e^{-j\omega t} - V_0^- e^{j\omega t}}{Z_0} \]

so we now have our phasor relations for \( I(t) \) and \( V_0(t) \)

In general \( V_0^+ \) can be complex and we can write it in polar form as:

\[
V_0^+ = |V_0^+| e^{j\beta_0^+} \quad V_0^- = |V_0^-| e^{-j\beta_0^-}
\]

there will depend upon the boundary conditions — we have waves

we can now go back to the time domain to get the instantaneous current and voltage:

\[
V_0(z,t) = \Re \left( V_0(t) e^{j\omega t} \right)
\]

\[
= \Re \left( \left( V_0^+ e^{-j\beta_0^+} + V_0^- e^{j\beta_0^-} \right) e^{j\omega t} \right)
\]

\[
= \Re \left( |V_0^+| e^{-j\beta_0^+} e^{j\omega t} + |V_0^-| e^{j\beta_0^-} e^{-j\omega t} \right)
\]

\[
= \Re \left( |V_0^+| e^{-j\beta_0^+} e^{j(wt - \beta_0^+ t)} + |V_0^-| e^{j\beta_0^-} e^{j(wt + \beta_0^- t)} \right)
\]

\[
= |V_0^+| e^{-j\beta_0^+} \cos(wt - \beta_0^+ t) + |V_0^-| e^{j\beta_0^-} \cos(wt + \beta_0^- t)
\]

there goes left \( \to \) right

\( \beta_0^+ \) towards \( +z \)

\( \beta_0^- \) towards \( -z \)

the phase velocity of the wave is

\[
U_p = \frac{\omega}{\beta}
\]

\[
\beta = \frac{2\pi}{\lambda_p}
\]

thus the wave direction is traveling in \( +z \)

\( \omega = 2\pi f \) the wave a plane wave
- the time varying signal sets up a traveling propagating wave on the transmission line which consists in general of a left → right going wave and a right → left going wave. The exact type of wave set up depends upon the "BOUNDARY" conditions and type of line.

Note: the fact that we can have waves propagating in both directions on a transmission line implies that under certain conditions we can produce a "standing wave" in one in which the nodes are indep. of time.

Example: let ϕ⁺ = ϕ⁻ = 0 and no attenuation ω = 0 (lossless)

Then ϕ(±t) = V₀[cos(ωt - βz) + cos(ωt + βz)]

where we have |V₀|² = |V₀| = |V₀|

now ω(A±B) = ωA ± ωB = V₀A ± V₀B

∴ with A = cos(ωt - βz) and B = cos(ωt + βz)

we get that

ωl(ωt - βz) + ωl(ωt + βz) = [ωlcosωtcosβz + ωlcosωtcosβz] =

[ωlcosωtcosβz - ωlcosωtcosβz] = 2ωlcosωtcosβz ← This does not a standing wave since the nodes - with nodes occur at same pt in z indep. of time (ie when βz = π/2, 3π/2, 5π/2 etc.

ωl(βz) = 0 indep. of t!

ωl(βz) = 0 indep. of t!
Derivation of Wave Equations

\[-\frac{d\tilde{V}(z)}{dz} = (R' + j\omega L')\tilde{I}(z),\]
\[-\frac{d\tilde{I}(z)}{dz} = (G' + j\omega C')\tilde{V}(z).\]

Combining the two equations leads to:

\[\frac{d^2\tilde{V}(z)}{dz^2} - (R' + j\omega L')(G' + j\omega C')\tilde{V}(z) = 0,\]

Second-order differential equation

\[\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')}. \quad (2.22)\]

\[\alpha = \Re(\gamma) = \Re\left(\sqrt{(R' + j\omega L')(G' + j\omega C')}\right) \quad (Np/m), \quad (2.25a)\]
\[\beta = \Im(\gamma) = \Im\left(\sqrt{(R' + j\omega L')(G' + j\omega C')}\right) \quad (\text{rad/m}). \quad (2.25b)\]
Solution of Wave Equations (cont.)

\[
\frac{d^2 \tilde{V}(z)}{dz^2} - \gamma^2 \tilde{V}(z) = 0, \quad (2.21)
\]

\[
\frac{d^2 \tilde{I}(z)}{dz^2} - \gamma^2 \tilde{I}(z) = 0. \quad (2.23)
\]

Proposed form of solution:

\[
\tilde{V}(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \quad (V),
\]

\[
\tilde{I}(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z} \quad (A).
\]

Using:

\[
- \frac{d\tilde{V}(z)}{dz} = (R' + j\omega L') \tilde{I}(z),
\]

It follows that:

\[
\tilde{I}(z) = \frac{\gamma}{R' + j\omega L'} \left[ V_0^+ e^{-\gamma z} - V_0^- e^{\gamma z} \right]
\]

Figure 2-9: In general, a transmission line can support two traveling waves, an incident wave [with voltage and current amplitudes \((V_0^+, I_0^+)\)] traveling along the \(+z\)-direction (towards the load) and a reflected wave [with \((V_0^-, I_0^-)\)] traveling along the \(-z\)-direction (towards the source).

Comparison of each term with the corresponding term in Eq. (2.26b) leads us to conclude that

\[
\frac{V_0^+}{I_0^+} = Z_0 = \frac{-V_0^-}{I_0^-}, \quad (2.28)
\]

where

\[
Z_0 = \frac{R' + j\omega L'}{\gamma} = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}} \quad (\Omega), \quad (2.29)
\]

is called the characteristic impedance of the line.
Solution of Wave Equations (cont.)

In general:

\[ V_0^+ = |V_0^+|e^{j\phi^+}, \]
\[ V_0^- = |V_0^-|e^{j\phi^-}. \]

\[ v(z, t) = \Re(\tilde{V}(z)e^{j \omega t}) \]
\[ = \Re \left[ (V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}) e^{j \omega t} \right] \]
\[ = \Re \left[ |V_0^+| e^{j \phi^+} e^{j \omega t} e^{-(\alpha + j\beta)z} \right. \]
\[ + \left. |V_0^-| e^{j \phi^-} e^{j \omega t} e^{(\alpha + j\beta)z} \right] \]
\[ = |V_0^+| e^{-\alpha z} \cos(\omega t - \beta z + \phi^+) \]
\[ + |V_0^-| e^{\alpha z} \cos(\omega t + \beta z + \phi^-) \]

The presence of two waves on the line propagating in opposite directions produces a standing wave.

Wave along +z because coefficients of \( t \) and \( z \) have opposite signs

Wave along –z because coefficients of \( t \) and \( z \) have the same sign
Example 2-1: Air Line

An air line is a transmission line in which air separates the two conductors, which renders \( G' = 0 \) because \( \sigma = 0 \). In addition, assume that the conductors are made of a material with high conductivity so that \( R' \approx 0 \). For an air line with a characteristic impedance of 50 \( \Omega \) and a phase constant of 20 rad/m at 700 MHz, find the line inductance \( L' \) and the line capacitance \( C' \).

Solution: The following quantities are given:

\[
Z_0 = 50 \, \Omega, \quad \beta = 20 \text{ rad/m},
\]
\[
f = 700 \text{ MHz} = 7 \times 10^8 \text{ Hz}.
\]

With \( R' = G' = 0 \), Eqs. (2.25b) and (2.29) reduce to

\[
\beta = \text{Im} \left[ \sqrt{(j \omega L')(j \omega C')} \right]
\]
\[
= \text{Im} \left( j \omega \sqrt{L'C'} \right) = \omega \sqrt{L'C'},
\]

\[
Z_0 = \sqrt{\frac{j \omega L'}{j \omega C'}} = \sqrt{\frac{L'}{C'}}.
\]

The ratio of \( \beta \) to \( Z_0 \) is

\[
\frac{\beta}{Z_0} = \omega C',
\]

or

\[
C' = \frac{\beta}{\omega Z_0} = \frac{20}{2\pi \times 7 \times 10^8 \times 50}
\]
\[
= 9.09 \times 10^{-11} \text{ (F/m)} = 90.9 \text{ (pF/m)}.
\]

From \( Z_0 = \sqrt{L'/C'} \), it follows that

\[
L' = Z_0^2 C'
\]
\[
= (50)^2 \times 90.9 \times 10^{-12}
\]
\[
= 2.27 \times 10^{-7} \text{ (H/m)} = 227 \text{ (nH/m)}.
\]
CD Module 2.1 Two-Wire Line The input data specifies the geometric and electrical parameters of a two-wire transmission line. The output includes the calculated values for the line parameters, characteristic impedance $Z_0$, and attenuation and phase constants, as well as plots of $Z_0$ as a function of $d$ and $D$. 

**Input**

- Wire Diameter $d = 1.7794$ [mm]
- Centers distance $D = 8.793$ [mm]
- Frequency $f = 1.0E9$ [Hz]
- Relative permittivity $\varepsilon_r = 2.3$
- Conductivity $\sigma = 0.0$ [S/m]
- Conductivity of the core $\sigma_c = 5.797E7$ [S/m]

**Output**

- $f = 1.0$ [GHz]
- $d = 1.7794$ [mm]
- $D = 8.793$ [mm]
- $Z_0 = 180.440373 - j0.046476$ [\Omega]
- $C' = 28.016183$ [pF/m]
- $L' = 912.171221$ [nH/m]
- $R' = 2.952465$ [\Omega/m]
- $G' = 0.0$ [S/m]
- $\lambda_0 = 0.3$ [m] in vacuum
- $\lambda = 0.1978$ [m] in guide
- $\alpha = 0.008181$ [Np/m]
- $\beta = 31.763075$ [rad/m]
CD Module 2.2 Coaxial Cable  Except for changing the geometric parameters to those of a coaxial transmission line, this module offers the same output information as Module 2.1.

**Input**
- Inner radius \( a = 3.034 \) [mm]
- Shield radius \( b = 14.7579 \) [mm]
- Frequency \( f = 5.38E9 \) [Hz]

**Output**
- Structure Data
  - \( a = 3.034 \) [mm] \( b / a = 4.86417 \)
  - \( b = 14.7579 \) [mm]
  - \( Z_0 = 62.584306 - j 0.0035419 \) [\( \Omega \)]
  - \( C' = 80.775048 \) [\( pF/m \)]
  - \( L' = 316.37933 \) [\( nH/m \)]
  - \( R' = 1.210519 \) [\( \Omega /m \)]
  - \( G' = 0.0 \) [\( S/m \)]
  - \( \lambda_0 = 5.5762 \) [cm] in vacuum
  - \( \lambda = 3.6768 \) [cm] in guide
  - \( \alpha = 0.009671 \) [\( Np/m \)]
  - \( \beta = 170.88534 \) [rad/m]