EE 135, Winter 2012

Reading: start Chapter 4, Electrostatics, Sections 1-7.

Homework #4, Part 1: chapter 3, prob.3.25,1,b,c,
Prob. 3.36, a,b,c, prob.3.54
The rest (from chapter 4) to be assigned Thursday
Homework set #4 due Thursday, Feb. 16, 2012.

Discussion Session: 7-8pm Wednesdays, Jack’s Lounge

GRADING POLICY:
2 midterm exams @25% each
Final Exam @ 35%
Homework @ 15%
4. ELECTROSTATICS

Applied EM by Ulaby, Michielssen and Ravaioli
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## Objectives

Upon learning the material presented in this chapter, you should be able to:

1. Evaluate the electric field and electric potential due to any distribution of electric charges.
2. Apply Gauss’s law.
3. Calculate the resistance $R$ of any shaped object, given the electric field at every point in its volume.
4. Describe the operational principles of resistive and capacitive sensors.
5. Calculate the capacitance of two-conductor configurations.
MAXWELL'S EQUATIONS
AND LIGHT
(AND HOW!)

Technically speaking, the electric and magnetic fields are VECTOR FIELDS - fields with a magnitude and direction at every point. To describe a vector field, you must specify how the field spreads out, or DIVERGES, and how it circles around, or CURLS. (DIVERGENCE AND CURL ARE MATHEMATICAL TERMS.)

In 1873, James Clerk MAXWELL wrote down four equations, which specify the curl and divergence of the electric and magnetic fields.

From: Chalmers University: Electromagnetics
XXV. On Physical Lines of Force. By J. C. Maxwell, Professor of Natural Philosophy in King's College, London.

PART I.—The Theory of Molecular Vortices applied to Magnetic Phenomena.

In all phenomena involving attractions or repulsions, or any forces depending on the relative position of bodies, we have to determine the magnitude and direction of the force which would act on a given body, if placed in a given position.

In the case of a body acted on by the gravitation of a sphere, this force is inversely as the square of the distance, and in a straight line to the centre of the sphere. In the case of two attracting spheres, or of a body not spherical, the magnitude and direction of the force vary according to more complicated laws. In electric and magnetic phenomena, the magnitude and direction of the resultant force at any point is the main subject of investigation. Suppose that the direction of the force at any point is known, then, if we draw a line so that in every part of its course it coincides in direction with the force at that point, this line may be called a *line of force*, since it indicates the direction of the force in every part of its course.

By drawing a sufficient number of lines of force, we may indicate the direction of the force in every part of the space in which it acts.

Thus if we strewn iron filings on paper near a magnet, each filing will be magnetized by induction, and the consecutive filings will unite by their opposite poles, so as to form fibres, and these fibres will indicate the direction of the lines of force.

The beautiful illustration of the presence of magnetic force afforded by this experiment, naturally tends to make us think of...
Light as an Electromagnetic Phenomena, 1864

James Clerk Maxwell (1831–1879)

Born 13 June 1831
Edinburgh, Scotland

Died 5 November 1879 (aged 48)
Cambridge, England

O.T.R.U.A.TOME. Sphere ds was done in the most general form in 1867. I have now lagged slightly from T & T and have the numerical values of \( \frac{M}{4} \) & S in 4 lines, thus varying T.7 value of \( \frac{M}{4} \).

You plan seems independent of T & T, & my. Publish!

I am busy supplying the physical necessaries of scientific life, within 12 Sereve Terrace, Cambridge. Probes have got ideas as grooves, corrugated plates, guiding arrows. If you desire home for criticism then:

\[
\int_{-1}^{1} \frac{(\sin x)^2}{x^2 + 1} \, dx = \frac{2}{2i+1} \left( \frac{1 + i}{1 - i} \right) \\
\text{Hence } \int_{-1}^{1} \frac{(\sin x)^2}{x^2 + 1} \, dx = \frac{2}{2i+1} \left( \frac{1 + i}{1 - i} \right) \\
\text{except when } \xi = 0 \text{ when } \int_{-1}^{1} \frac{(\sin x)^2}{x^2 + 1} \, dx = \frac{2i}{2i+1} \left( \frac{1 + i}{1 - i} \right)
\]
Maxwell’s Equations

\[ \nabla \cdot \mathbf{D} = \rho_v, \]
\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \]
\[ \nabla \cdot \mathbf{B} = 0, \]
\[ \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}. \]

Under **static** conditions, none of the quantities appearing in Maxwell’s equations are functions of time (i.e., \( \partial / \partial t = 0 \)). This happens when all charges are permanently fixed in space, or, if they move, they do so at a steady rate so that \( \rho_v \) and \( \mathbf{J} \) are constant in time. Under these circumstances, the time derivatives of \( \mathbf{B} \) and \( \mathbf{D} \) in Eqs. (4.1b) and (4.1d) vanish, and Maxwell’s equations reduce to

**Electrostatics**

\[ \nabla \cdot \mathbf{D} = \rho_v, \quad (4.2a) \]
\[ \nabla \times \mathbf{E} = 0. \quad (4.2b) \]

**Magnetostatics**

\[ \nabla \cdot \mathbf{B} = 0, \quad (4.3a) \]
\[ \nabla \times \mathbf{H} = \mathbf{J}. \quad (4.3b) \]

*Electric and magnetic fields become decoupled under static conditions.*
\[ \nabla \cdot \mathbf{D} = \rho_v \]

Maxwell’s First Equation is Gauss’s Law. It says that electric field lines diverge from positive charges and converge to negative charges.

\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \]

The second equation is Faraday’s Law: Electric field lines curl around changing magnetic fields. Changing magnetic fields induce electric fields.

From Chalmers.E&M
\[ \nabla \cdot \mathbf{B} = 0 \]

\[ \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \]

The third equation says that magnetic fields never diverge or converge. They always go in closed curves.

Finally, the last equation says that magnetic field lines \textbf{curl} around electric currents. We have seen that a magnetic field circles around a conducting wire.

AND HERE MAXWELL HAD A CRITICAL BRAINSTORM! (AN ELECTRICAL STORM, OF COURSE!)
Charge Distributions

Volume charge density:

\[
\rho_v = \lim_{\Delta V \to 0} \frac{\Delta q}{\Delta V} = \frac{dq}{dV} \quad (\text{C/m}^3)
\]

Total Charge in a Volume

\[
Q = \int_V \rho_v \, dV \quad (\text{C})
\]

Surface and Line Charge Densities

\[
\rho_s = \lim_{\Delta s \to 0} \frac{\Delta q}{\Delta s} = \frac{dq}{ds} \quad (\text{C/m}^2)
\]

\[
\rho_l = \lim_{\Delta l \to 0} \frac{\Delta q}{\Delta l} = \frac{dq}{dl} \quad (\text{C/m})
\]
The amount of charge that crosses the tube’s cross-sectional surface $\Delta s'$ in time $\Delta t$ is therefore

$$\Delta q' = \rho_v \Delta V = \rho_v \Delta l \Delta s' = \rho_v u \Delta s' \Delta t. \quad (4.8)$$

For a surface with any orientation:

$$\Delta q = \rho_v u \cdot \Delta s \Delta t, \quad (4.9)$$

where $\Delta s = \hat{n} \Delta s$ and the corresponding total current flowing in the tube is

$$\Delta I = \frac{\Delta q}{\Delta t} = \rho_v u \cdot \Delta s = J \cdot \Delta s, \quad (4.10)$$

where

$$J = \rho_v u \quad (A/m^2) \quad (4.11)$$

$J$ is called the current density.

Figure 4-2: Charges with velocity $u$ moving through a cross section $\Delta s'$ in (a) and $\Delta s$ in (b).

When a current is due to the actual movement of electrically charged matter, it is called a convection current, and $J$ is called a convection current density.
When a current is due to the movement of charged particles relative to their host material, \( J \) is called a *conduction current density*.

This movement of electrons from atom to atom constitutes a *conduction current*. The electrons that emerge from the wire are not necessarily the same electrons that entered the wire at the other end.

Conduction current, which is discussed in more detail in Section 4-6, obeys Ohm’s law, whereas convection current does not.
Charles Augustin de Coulomb

Diagram describing the basic mechanism of Coulomb's law. Like charges repel each other, opposite charges attract each other.

\[ |F_{Q-q}| = |F_{q-Q}| = k \frac{|q| \times |Q|}{r^2} \]
Coulomb's experiment with the torsion balance (1777)
How the Torsion Balance Works?

The torsion balance consists of a bar suspended from its middle by a thin fiber. The fiber acts as a very weak torsion spring. If an unknown force is applied at right angles to the ends of the bar, the bar will rotate, twisting the fiber, until it reaches an equilibrium where the twisting force or torque of the fiber balances the applied force. Then the magnitude of the force is proportional to the angle of the bar. The sensitivity of the instrument comes from the weak spring constant of the fiber, so a very weak force causes a large rotation of the bar.

In Coulomb's experiment, the torsion balance was an insulating rod with a metal-coated ball attached to one end, suspended by a silk thread. The ball was charged with a known charge of static electricity, and a second charged ball of the same polarity was brought near it. The two charged balls repelled one another, twisting the fiber through a certain angle, which could be read from a scale on the instrument. By knowing how much force it took to twist the fiber through a given angle, Coulomb was able to calculate the force between the balls. Determining the force for different charges and different separations between the balls, he showed that it followed an inverse-square proportionality law, now known as Coulomb's law.

Coulomb, 1777: electrostatic force ------Cavendish, 1798, gravitational force
Coulomb’s Law

Electric field at point P due to single charge

\[ E = \hat{R} \frac{q}{4\pi \varepsilon R^2} \quad \text{(V/m)} \]

Electric force on a test charge placed at P

\[ F = q' E \quad \text{(N)} \]

Electric flux density \( \mathbf{D} \)

\[ \mathbf{D} = \varepsilon \mathbf{E} \]

\[ \varepsilon = \varepsilon_r \varepsilon_0, \]

\[ \varepsilon_0 = 8.85 \times 10^{-12} \approx (1/36\pi) \times 10^{-9} \quad \text{(F/m)} \]

*If \( \varepsilon \) is independent of the magnitude of \( \mathbf{E} \), then the material is said to be linear because \( \mathbf{D} \) and \( \mathbf{E} \) are related linearly, and if it is independent of the direction of \( \mathbf{E} \), the material is said to be isotropic.*
Electric Field Due to 2 Charges

with $R$, the distance between $q_1$ and $P$, replaced with $|\mathbf{R} - \mathbf{R}_1|$ and the unit vector $\hat{\mathbf{R}}$ replaced with $(\mathbf{R} - \mathbf{R}_1)/|\mathbf{R} - \mathbf{R}_1|$. Thus,

$$E_1 = \frac{q_1(\mathbf{R} - \mathbf{R}_1)}{4\pi \varepsilon |\mathbf{R} - \mathbf{R}_1|^3} \text{ (V/m)}. \tag{4.17a}$$

Similarly, the electric field at $P$ due to $q_2$ alone is

$$E_2 = \frac{q_2(\mathbf{R} - \mathbf{R}_2)}{4\pi \varepsilon |\mathbf{R} - \mathbf{R}_2|^3} \text{ (V/m)}. \tag{4.17b}$$

*The electric field obeys the principle of linear superposition.*

Hence, the total electric field $\mathbf{E}$ at $P$ due to $q_1$ and $q_2$ is

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$$

$$= \frac{1}{4\pi \varepsilon} \left[ \frac{q_1(\mathbf{R} - \mathbf{R}_1)}{|\mathbf{R} - \mathbf{R}_1|^3} + \frac{q_2(\mathbf{R} - \mathbf{R}_2)}{|\mathbf{R} - \mathbf{R}_2|^3} \right]. \tag{4.18}$$
Two Aperture Electron Interference Pattern Using 100 KeV Electrons
Electric Field due to Multiple Charges

Example 4-3: Electric Field Due to Two Point Charges

Two point charges with \( q_1 = 2 \times 10^{-5} \) C and \( q_2 = -4 \times 10^{-5} \) C are located in free space at points with Cartesian coordinates \((1, 3, -1)\) and \((-3, 1, -2)\), respectively. Find (a) the electric field \( E \) at \((3, 1, -2)\) and (b) the force on a \(8 \times 10^{-5} \) C charge located at that point. All distances are in meters.

**Solution:**

(a) From Eq. (4.18), the electric field \( E \) with \( \varepsilon = \varepsilon_0 \) (free space) is

\[
E = \frac{1}{4\pi \varepsilon_0} \left[ q_1 \frac{(R - R_1)}{|R - R_1|^3} + q_2 \frac{(R - R_2)}{|R - R_2|^3} \right] \quad \text{(V/m)}.
\]

The vectors \( R_1, R_2, \) and \( R \) are

\[
R_1 = \hat{x} + \hat{y}3 - \hat{z},
\]

\[
R_2 = -\hat{x}3 + \hat{y} - \hat{z}2,
\]

\[
R = \hat{x}3 + \hat{y} - \hat{z}2.
\]

Hence,

\[
E = \frac{1}{4\pi \varepsilon_0} \left[ \frac{2(\hat{x}2 - \hat{y}2 - \hat{z})}{27} - \frac{4(\hat{x}6)}{216} \right] \times 10^{-5}
\]

\[
= \frac{\hat{x} - \hat{y}4 - \hat{z}2}{108\pi \varepsilon_0} \times 10^{-5} \quad \text{(V/m)}.
\]

(b)

\[
F = q_3E = 8 \times 10^{-5} \times \frac{\hat{x} - \hat{y}4 - \hat{z}2}{108\pi \varepsilon_0} \times 10^{-5}
\]

\[
= \frac{\hat{x}2 - \hat{y}8 - \hat{z}4}{27\pi \varepsilon_0} \times 10^{-10} \quad \text{(N)}.
\]
Electric Field Due to Charge Distributions

Field due to:

A differential amount of charge $dq = \rho_v \, dV'$ contained in a differential volume $dV'$ is

$$dE = \hat{R}' \frac{dq}{4\pi \varepsilon R'^2} = \hat{R}' \frac{\rho_v \, dV'}{4\pi \varepsilon R'^2}, \quad (4.20)$$

$$E = \int_{V'} dE = \frac{1}{4\pi \varepsilon} \int_{V'} \hat{R}' \frac{\rho_v \, dV'}{R'^2}$$

(volume distribution). \hspace{1cm} (4.21a)

$$E = \frac{1}{4\pi \varepsilon} \int_{S'} \hat{R}' \frac{\rho_s \, ds'}{R'^2}$$

(surface distribution), \hspace{1cm} (4.21b)

$$E = \frac{1}{4\pi \varepsilon} \int_{l'} \hat{R}' \frac{\rho_l \, dl'}{R'^2}$$

(line distribution). \hspace{1cm} (4.21c)
**Example 4-4: Electric Field of a Ring of Charge**

A ring of charge of radius $b$ is characterized by a uniform line charge density of positive polarity $\rho_\ell$. The ring resides in free space and is positioned in the $x$-$y$ plane as shown in Fig. 4-6. Determine the electric field intensity $\mathbf{E}$ at a point $P = (0, 0, h)$ along the axis of the ring at a distance $h$ from its center.

**Solution:** We start by considering the electric field generated by a differential ring segment with cylindrical coordinates $(b, \phi, 0)$ in Fig. 4-6(a). The segment has length $dl = b \, d\phi$ and contains charge $dq = \rho_\ell \, dl = \rho_\ell b \, d\phi$. The distance vector $\mathbf{R}'_1$ from segment 1 to point $P = (0, 0, h)$ is

$$\mathbf{R}'_1 = -\hat{r}b + \hat{z}h,$$

from which it follows that

$$R'_1 = |\mathbf{R}'_1| = \sqrt{b^2 + h^2}, \quad \hat{\mathbf{R}}'_1 = \frac{\mathbf{R}'_1}{R'_1} = \frac{-\hat{r}b + \hat{z}h}{\sqrt{b^2 + h^2}}.$$

The electric field at $P = (0, 0, h)$ due to the charge in segment 1 therefore is

$$dE_1 = \frac{1}{4\pi \varepsilon_0} \frac{\rho_\ell \, dl}{R'_1^2} = \frac{\rho_\ell b}{4\pi \varepsilon_0} \frac{(-\hat{r}b + \hat{z}h)}{(b^2 + h^2)^{3/2}} \, d\phi.$$

![Figure 4-6: Ring of charge with line density $\rho_\ell$. (a) The field $dE_1$ due to infinitesimal segment 1 and (b) the fields $dE_1$ and $dE_2$ due to segments at diametrically opposite locations (Example 4-4).](image)
\[
\frac{dE_1}{dE_1} = \frac{1}{4\pi \varepsilon_0} \hat{R}' \frac{\rho_\ell \, dl}{R_1^2} = \frac{\rho_\ell b}{4\pi \varepsilon_0} \frac{(-\hat{b} + \hat{h})}{(b^2 + h^2)^{3/2}} \, d\phi.
\]

The field \(dE_1\) has component \(dE_{1r}\) along \(-\hat{r}\) and component \(dE_{1z}\) along \(\hat{z}\). From symmetry considerations, the field \(dE_2\) generated by differential segment 2 in Fig. 4-6(b), which is located diametrically opposite to segment 1, is identical to \(dE_1\) except that the \(\hat{r}\)-component of \(dE_2\) is opposite that of \(dE_1\). Hence, the \(\hat{r}\)-components in the sum cancel and the \(\hat{z}\)-contributions add. The sum of the two contributions is

\[
dE = dE_1 + dE_2 = \hat{z} \frac{\rho_\ell bh}{2\pi \varepsilon_0 (b^2 + h^2)^{3/2}} \, d\phi.
\]

Since for every ring segment in the semicircle defined over the azimuthal range \(0 \leq \phi \leq \pi\) (the right-hand half of the circular ring) there is a corresponding segment located diametrically opposite at \((\phi + \pi)\), we can obtain the total field generated by the ring by integrating Eq. (4.22) over a semicircle as

\[
E = \hat{z} \frac{\rho_\ell bh}{2\pi \varepsilon_0 (b^2 + h^2)^{3/2}} \int_0^\pi d\phi
\]

\[
= \hat{z} \frac{\rho_\ell bh}{2\pi \varepsilon_0 (b^2 + h^2)^{3/2}} \frac{h}{4\pi \varepsilon_0 (b^2 + h^2)^{3/2}} Q,
\]

where \(Q = 2\pi b\rho_\ell\) is the total charge on the ring.

**Figure 4-6:** Ring of charge with line density \(\rho_\ell\). (a) The field \(dE_1\) due to infinitesimal segment 1 and (b) the fields \(dE_1\) and \(dE_2\) due to segments at diametrically opposite locations (Example 4-4).
Example 4-5: Electric Field of a Circular Disk of Charge

Find the electric field at point $P$ with Cartesian coordinates $(0, 0, h)$ due to a circular disk of radius $a$ and uniform charge density $\rho_s$ residing in the $x$–$y$ plane (Fig. 4-7). Also, evaluate $\mathbf{E}$ due to an infinite sheet of charge density $\rho_s$ by letting $a \to \infty$.

Solution: Building on the expression obtained in Example 4-4 for the on-axis electric field due to a circular ring of charge, we can determine the field due to the circular disk by treating the disk as a set of concentric rings. A ring of radius $r$ and width $dr$ has an area $ds = 2\pi r \, dr$ and contains charge $dq = \rho_s \, ds = 2\pi \rho_s r \, dr$. Upon using this expression in Eq. (4.23) and also replacing $b$ with $r$, we obtain the following expression for the field due to the ring:

$$d\mathbf{E} = \hat{\mathbf{z}} \frac{h}{4\pi \varepsilon_0 (r^2 + h^2)^{3/2}} (2\pi \rho_s r \, dr).$$

Figure 4-7: Circular disk of charge with surface charge density $\rho_s$. The electric field at $P = (0, 0, h)$ points along the $z$-direction (Example 4-5).
Example 4-5 cont.

The total field at \( P \) is obtained by integrating the expression over the limits \( r = 0 \) to \( r = a \):

\[
E = \hat{\mathbf{z}} \frac{\rho_s h}{2\varepsilon_0} \int_0^a \frac{r \, dr}{(r^2 + h^2)^{3/2}} = \pm \hat{\mathbf{z}} \frac{\rho_s}{2\varepsilon_0} \left[ 1 - \frac{|h|}{\sqrt{a^2 + h^2}} \right],
\]

with the plus sign for \( h > 0 \) (\( P \) above the disk) and the minus sign when \( h < 0 \) (\( P \) below the disk).

For an infinite sheet of charge with \( a = \infty \),

\[
E = \pm \hat{\mathbf{z}} \frac{\rho_s}{2\varepsilon_0} \quad \text{(infinite sheet of charge).}
\]

We note that for an infinite sheet of charge \( \mathbf{E} \) is the same at all points above the \( x-y \) plane, and a similar statement applies for points below the \( x-y \) plane.

Figure 4-7: Circular disk of charge with surface charge density \( \rho_s \). The electric field at \( P = (0, 0, h) \) points along the \( z \)-direction (Example 4-5).
Start with the first law

\[ \nabla \cdot \mathbf{D} = \rho_v \]

Maxwell’s first equation is Gauss’s law. It says that electric field lines diverge from positive charges and converge to negative charges.

Carl Friedrich Gauss (1777–1855), painted by Christian Albrecht Jensen

**Born**
30 April 1777
Braunschweig, Duchy of Brunswick-Wolfenbüttel, Holy Roman Empire

**Died**
23 February 1855
(aged 77)
Göttingen, Kingdom of Hanover

**Residence**
Kingdom of Hanover

**Nationality**
German

**Fields**
Mathematics and Physics
Gauss’s Law

\[ \nabla \cdot \mathbf{D} = \rho_v \]

(Differential form of Gauss’s law),

\[ \int_V \nabla \cdot \mathbf{D} \, dV = \int_V \rho_v \, dV = Q \]

Application of the divergence theorem gives:

\[ \int_V \nabla \cdot \mathbf{D} \, dV = \oint_S \mathbf{D} \cdot d\mathbf{s}. \] (4.28)

Comparison of Eq. (4.27) with Eq. (4.28) leads to

\[ \oint_S \mathbf{D} \cdot d\mathbf{s} = Q \] (4.29)

(Integral form of Gauss’s law).

The integral form of Gauss’s law is illustrated diagrammatically in Fig. 4-8; for each differential surface element \( d\mathbf{s} \), \( \mathbf{D} \cdot d\mathbf{s} \) is the electric field flux flowing outward of \( V \) through \( d\mathbf{s} \), and the total flux through surface \( S \) equals the enclosed charge \( Q \). The surface \( S \) is called a Gaussian surface.

**Figure 4-8:** The integral form of Gauss’s law states that the outward flux of \( \mathbf{D} \) through a surface is proportional to the enclosed charge \( Q \).
Applying Gauss’s Law

\[ \oint \mathbf{D} \cdot d\mathbf{s} = Q \]  
(4.29)

(Integral form of Gauss’s law).

**Example 4-6: Electric Field of an Infinite Line Charge**

Use Gauss’s law to obtain an expression for \( \mathbf{E} \) due to an infinitely long line with uniform charge density \( \rho \) that resides along the \( z \)-axis in free space.

Construct an imaginary Gaussian cylinder of radius \( r \) and height \( h \):

\[ \int \int_{z=0}^{h} \hat{r} D_r \cdot \hat{r} \, d\phi \, dz = \rho \ell h \]

or

\[ 2\pi h D_r r = \rho \ell h, \]

which yields

\[ \mathbf{E} = \frac{\mathbf{D}}{\varepsilon_0} = \hat{r} \frac{D_r}{\varepsilon_0} = \hat{r} \frac{\rho \ell}{2\pi \varepsilon_0 r} \]  
(4.33)

(infinite line charge).
Electric Field due to Infinite Line Charge
(using Coulomb's law)

\[ \mathbf{dE} = \hat{r} \, dE_r \]

by symmetry, the \( \hat{z} \) component cancels, only left with \( \hat{r} \) component.

Coulomb's law: \[ dE_r = \frac{dq \, \cos \omega}{4\pi \varepsilon (r^2 + z^2)} \]

where \( dq = \rho \, dr \)

by geometry:
\[ z = r \tan \alpha \]
\[ dz = r \sec^2 \alpha \, d\alpha \]
\[ \sqrt{r^2 + z^2} = r \sec \alpha \]

\[ dE_r = \left[ \frac{\rho \, r \sec^2 \alpha \, d\alpha}{4\pi \varepsilon r^2 \sec^2 \alpha} \right] \cos \omega = \frac{\rho \, \cos \omega \, d\alpha}{4\pi \varepsilon r^2} \]
\[ E_r = \frac{m_2}{4\pi \varepsilon} \int_{\alpha_1}^{\alpha_2} \frac{\rho \, \cos \omega \, d\alpha}{r^2} = \frac{m_2 \rho}{4\pi \varepsilon} \left[ \sin \alpha \right]_{\alpha_1}^{\alpha_2} \]
\[ \therefore \sqrt{\frac{m_2 \rho}{4\pi \varepsilon}} \text{ } \text{same as Gauss' law} \]
The term “voltage” is short for “voltage potential” and synonymous with electric potential.

Minimum force needed to move charge against \( E \) field:

\[
F_{\text{ext}} = -F_e = -qE. \tag{4.34}
\]

The work done, or energy expended, in moving any object a vector differential distance \( dl \) while exerting a force \( F_{\text{ext}} \) is

\[
dW = F_{\text{ext}} \cdot dl = -qE \cdot dl \quad (\text{J}). \tag{4.35}
\]

Work, or energy, is measured in joules (J). If the charge is moved a distance \( dy \) along \( \hat{y} \), then

\[
dW = -q(-\hat{y}E) \cdot \hat{y} dy = qE \, dy. \tag{4.36}
\]

The differential electric potential energy \( dW \) per unit charge is called the \textit{differential electric potential} (or differential voltage) \( dV \). That is,

\[
dV = \frac{dW}{q} = -E \cdot dl \quad (\text{J/C or V}). \tag{4.37}
\]

\[+ \text{ charge, } q\]
\[E = -yE, \text{ -y direction}\]
Electric Scalar Potential

Figure 4-12: In electrostatics, the potential difference between $P_2$ and $P_1$ is the same irrespective of the path used for calculating the line integral of the electric field between them.

\[ \int_{P_1}^{P_2} dV = - \int_{P_1}^{P_2} \mathbf{E} \cdot d\mathbf{l}, \]

\[ V_{21} = V_2 - V_1 = - \int_{P_1}^{P_2} \mathbf{E} \cdot d\mathbf{l}, \quad (4.39) \]

\[ \int_{C} \mathbf{E} \cdot d\mathbf{l} = 0 \quad \text{(Electrostatics).} \quad (4.40) \]

A vector field whose line integral along any closed path is zero is called a conservative or an irrotational field. Hence, the electrostatic field $\mathbf{E}$ is conservative.
Conservative Property of Electrostatic Fields

Maxwell's 2\textsuperscript{nd} Law:
\[ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = 0 \text{ for electrostatics} \]

\[ \therefore \int_{C} (\nabla \times \vec{E}) \cdot d\vec{s} = \int_{C} \vec{E} \cdot d\vec{l} = 0 \]

Stoke's Theorem

\[ \therefore \text{line integral of } \vec{E} \text{ around any closed contour is zero, so potential difference between 2 pts. is independent of path} \]
In electric circuits, we usually select a convenient node that we call ground and assign it zero reference voltage. In free space and material media, we choose infinity as reference with \( V = 0 \). Hence, at a point \( P \):

\[
V_2 - V_1 = - \int_{P_1}^{P_2} \mathbf{E} \cdot d\mathbf{l}, \quad (4.39)
\]

For a point charge, \( V \) at range \( R \) is:

\[
V = - \int_{\infty}^{R} \left( \hat{\mathbf{r}} \frac{q}{4\pi \varepsilon R^2} \right) \cdot \hat{\mathbf{r}} \, dR = \frac{q}{4\pi \varepsilon R} \quad (V). \quad (4.45)
\]

For continuous charge distributions:

\[
V = \frac{1}{4\pi \varepsilon} \int_{V'} \frac{\rho_v}{R} \, dV' \quad \text{(volume distribution),} \quad (4.48a)
\]

\[
V = \frac{1}{4\pi \varepsilon} \int_{S'} \frac{\rho_s}{R} \, ds' \quad \text{(surface distribution),} \quad (4.48b)
\]

\[
V = \frac{1}{4\pi \varepsilon} \int_{l'} \frac{\rho_l}{R} \, dl' \quad \text{(line distribution).} \quad (4.48c)
\]
Relating $\mathbf{E}$ to $V$

\[ dV = -\mathbf{E} \cdot d\mathbf{l}. \quad (4.49) \]

For a scalar function $V$, Eq. (3.73) gives

\[ dV = \nabla V \cdot d\mathbf{l}, \quad (4.50) \]

where $\nabla V$ is the gradient of $V$. Comparison of Eq. (4.49) with Eq. (4.50) leads to

\[ \mathbf{E} = -\nabla V. \quad (4.51) \]

This differential relationship between $V$ and $\mathbf{E}$ allows us to determine $\mathbf{E}$ for any charge distribution by first calculating $V$ and then taking the negative gradient of $V$ to find $\mathbf{E}$. 
Example 4-7: Electric Field of an Electric Dipole

Solution: To simplify the derivation, we align the dipole along the z-axis and center it at the origin [Fig. 4-13(a)]. For the two charges shown in Fig. 4-13(a), application of Eq. (4.47) gives

\[ V = \frac{1}{4\pi \varepsilon_0} \left( \frac{q}{R_1} + \frac{-q}{R_2} \right) = \frac{q}{4\pi \varepsilon_0} \left( \frac{R_2 - R_1}{R_1 R_2} \right). \]

Since \( d \ll R \), the lines labeled \( R_1 \) and \( R_2 \) in Fig. 4-13(a) are approximately parallel to each other, in which case the following approximations apply:

\[ R_2 - R_1 \approx d \cos \theta , \quad R_1 R_2 \approx R^2. \]

Hence,

\[ V = \frac{qd \cos \theta}{4\pi \varepsilon_0 R^2} . \] (4.52)

(b) Electric-field pattern
Example 4-7: Electric Field of an Electric Dipole (cont.)

\[ qd \cos \theta = qd \cdot \hat{R} = p \cdot \hat{R}, \]

where \( p = qd \) is called the dipole moment. Using Eq. (4.53) in Eq. (4.52) then gives

\[ V = \frac{p \cdot \hat{R}}{4\pi \varepsilon_0 R^2} \quad \text{(electric dipole)}. \quad (4.54) \]

In spherical coordinates, Eq. (4.51) is given by

\[ E = -\nabla V \]
\[ = -\left( \hat{R} \frac{\partial V}{\partial R} + \hat{\theta} \frac{1}{R} \frac{\partial V}{\partial \theta} + \hat{\phi} \frac{1}{R \sin \theta} \frac{\partial V}{\partial \phi} \right), \quad (4.55) \]

\[ E = \frac{qd}{4\pi \varepsilon_0 R^3} \left( \hat{R} 2 \cos \theta + \hat{\theta} \sin \theta \right) \quad \text{(V/m)}. \]
Poisson’s & Laplace’s Equations

With \( \mathbf{D} = \varepsilon \mathbf{E} \), the differential form of Gauss’s law given by Eq. (4.26) may be cast as

\[
\nabla \cdot \mathbf{E} = \frac{\rho_v}{\varepsilon}.
\]  

(4.57)

Inserting Eq. (4.51) in Eq. (4.57) gives

\[
\nabla \cdot (\nabla V) = -\frac{\rho_v}{\varepsilon}.
\]  

(4.58)

Given Eq. (3.110) for the Laplacian of a scalar function \( V \),

\[
\nabla^2 V = \nabla \cdot (\nabla V) = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}.
\]  

(4.59)

Eq. (4.58) can be cast in the abbreviated form

\[
\nabla^2 V = -\frac{\rho_v}{\varepsilon} \quad \text{(Poisson’s equation).}
\]  

(4.60)

This is known as Poisson’s equation. For a volume \( V \) containing a volume charge density distribution \( \rho_v \), the solution for \( V \) derived previously and expressed by Eq. (4.48a) as

\[
V = \frac{1}{4\pi \varepsilon} \int \frac{\rho_v}{R'} dV'
\]  

(4.61)

In the absence of charges:

\[
\nabla^2 V = 0 \quad \text{(Laplace’s equation),}
\]
CD Module 4.1 Fields due to Charges For any group of point charges, this module calculates and displays the electric field \( \mathbf{E} \) and potential \( V \) across a 2-D grid. The user can specify the locations, magnitudes and polarities of the charges.
Conduction Current

The conductivity of a material is a measure of how easily electrons can travel through the material under the influence of an externally applied electric field.

Conduction current density:

\[ \mathbf{J} = \sigma \mathbf{E} \quad (\text{A/m}^2) \quad \text{(Ohm’s law)} \]

A perfect dielectric is a material with \( \sigma = 0 \). In contrast, a perfect conductor is a material with \( \sigma = \infty \). Some materials, called superconductors, exhibit such a behavior.

Note how wide the range is, over 24 orders of magnitude

<table>
<thead>
<tr>
<th>Material</th>
<th>Conductivity, ( \sigma ) (S/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Conductors</strong></td>
<td></td>
</tr>
<tr>
<td>Silver</td>
<td>( 6.2 \times 10^7 )</td>
</tr>
<tr>
<td>Copper</td>
<td>( 5.8 \times 10^7 )</td>
</tr>
<tr>
<td>Gold</td>
<td>( 4.1 \times 10^7 )</td>
</tr>
<tr>
<td>Aluminum</td>
<td>( 3.5 \times 10^7 )</td>
</tr>
<tr>
<td>Iron</td>
<td>( 10^7 )</td>
</tr>
<tr>
<td>Mercury</td>
<td>( 10^6 )</td>
</tr>
<tr>
<td>Carbon</td>
<td>( 3 \times 10^4 )</td>
</tr>
<tr>
<td><strong>Semiconductors</strong></td>
<td></td>
</tr>
<tr>
<td>Pure germanium</td>
<td>2.2</td>
</tr>
<tr>
<td>Pure silicon</td>
<td>( 4.4 \times 10^{-4} )</td>
</tr>
<tr>
<td><strong>Insulators</strong></td>
<td></td>
</tr>
<tr>
<td>Glass</td>
<td>( 10^{-12} )</td>
</tr>
<tr>
<td>Paraffin</td>
<td>( 10^{-15} )</td>
</tr>
<tr>
<td>Mica</td>
<td>( 10^{-15} )</td>
</tr>
<tr>
<td>Fused quartz</td>
<td>( 10^{-17} )</td>
</tr>
</tbody>
</table>
Conductivity

\[
\sigma = - \rho_{ve} \mu_e + \rho_{vh} \mu_h
= (N_e \mu_e + N_h \mu_h)e \quad \text{(S/m)} \quad \text{(semiconductor)}, \tag{4.67a}
\]

and its unit is siemens per meter (S/m). For a good conductor, \(N_h \mu_h \ll N_e \mu_e\), and Eq. (4.67a) reduces to

\[
\sigma = - \rho_{ve} \mu_e = N_e \mu_e e \quad \text{(S/m)} \quad \text{(conductor)}. \tag{4.67b}
\]

In view of Eq. (4.66), in a perfect dielectric with \(\sigma = 0\), \(J = 0\) regardless of \(E\), and in a perfect conductor with \(\sigma = \infty\), \(E = J/\sigma = 0\) regardless of \(J\).

That is,

\[
J = \sigma E \quad \text{(A/m}^2\text{)} \quad \text{(Ohm’s law),}
\]

Perfect dielectric: \(J = 0\).
Perfect conductor: \(E = 0\).
Example 4-8: Conduction Current in a Copper Wire

A 2-mm-diameter copper wire with conductivity of $5.8 \times 10^7$ S/m and electron mobility of 0.0032 (m$^2$/V·s) is subjected to an electric field of 20 (mV/m). Find (a) the volume charge density of the free electrons, (b) the current density, (c) the current flowing in the wire, (d) the electron drift velocity, and (e) the volume density of the free electrons.

Remember from last slide

\[
\sigma = -\rho_v \mu_e + \rho_h \mu_h \\
= (N_e \mu_e + N_h \mu_h) e \quad (S/m) \quad \text{(semiconductor),} \\
(4.67a)
\]

and its unit is siemens per meter (S/m). For a good conductor, $N_h \mu_h \ll N_e \mu_e$, and Eq. (4.67a) reduces to

\[
\sigma = -\rho_v \mu_e = N_e \mu_e e \quad (S/m) \\
\text{(conductor).} \quad (4.67b)
\]
Example 4.8 (cont)

Solution:

(a)
\[ \rho_{ve} = -\frac{\sigma}{\mu_e} = -\frac{5.8 \times 10^7}{0.0032} = -1.81 \times 10^{10} \text{ (C/m}^3\text{).} \]

(b)
\[ J = \sigma E = 5.8 \times 10^7 \times 20 \times 10^{-3} = 1.16 \times 10^6 \text{ (A/m}^2\text{).} \]

(c)
\[ I = J A = J \left( \frac{\pi d^2}{4} \right) = 1.16 \times 10^6 \left( \frac{\pi \times 4 \times 10^{-6}}{4} \right) = 3.64 \text{ A.} \]

(d)
\[ u_e = -\mu_e E = -0.0032 \times 20 \times 10^{-3} = -6.4 \times 10^{-5} \text{ m/s.} \]

The minus sign indicates that \( u_e \) is in the opposite direction of \( E \).

(e)
\[ N_e = -\frac{\rho_{ve}}{e} = -\frac{1.81 \times 10^{10}}{1.6 \times 10^{-19}} = 1.13 \times 10^{29} \text{ electrons/m}^3. \]
Ohm’s law: $R = \frac{V}{I}$

Georg Simon Ohm

Born
16 March 1789
Erlangen, Germany

Died
6 July 1854 (aged 65)
Munich, Germany
Resistance

Longitudinal Resistor

\[ V = V_1 - V_2 = -\int_{x_2}^{x_1} \mathbf{E} \cdot d\mathbf{l} \]
\[ = -\int_{x_2}^{x_1} \hat{x} E_x \cdot \hat{x} \, dl = E_x l \quad (V). \quad (4.68) \]

Using Eq. (4.63), the current flowing through the cross section \( A \) at \( x_2 \) is

\[ I = \int_{A} \mathbf{J} \cdot d\mathbf{s} = \int_{A} \sigma \mathbf{E} \cdot d\mathbf{s} = \sigma E_x A \quad (A). \quad (4.69) \]

From \( R = V/I \), the ratio of Eq. (4.68) to Eq. (4.69) gives

\[ R = \frac{I}{\sigma A} \quad (\Omega). \quad (4.70) \]

For any conductor:

\[ R = \frac{V}{I} = \frac{-\int_{l} \mathbf{E} \cdot d\mathbf{l}}{\int_{s} \mathbf{J} \cdot d\mathbf{s}} = \frac{-\int_{l} \mathbf{E} \cdot d\mathbf{l}}{\int_{s} \sigma \mathbf{E} \cdot d\mathbf{s}}. \]

\[ G = 1/R \]
Table 2-1: Transmission-line parameters \( R', L', G', \) and \( C' \) for three types of lines.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Coaxial</th>
<th>Two-Wire</th>
<th>Parallel-Plate</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R' )</td>
<td>( \frac{R_s}{2\pi} \left( \frac{1}{a} + \frac{1}{b} \right) )</td>
<td>( \frac{2R_s}{\pi d} )</td>
<td>( \frac{2R_s}{w} )</td>
<td>( \Omega/m )</td>
</tr>
<tr>
<td>( L' )</td>
<td>( \frac{\mu}{2\pi} \ln(b/a) )</td>
<td>( \frac{\mu}{\pi} \ln \left[ \left( \frac{D}{d} \right) + \sqrt{\left( \frac{D}{d} \right)^2 - 1} \right] )</td>
<td>( \frac{\mu h}{w} )</td>
<td>( \text{H/m} )</td>
</tr>
<tr>
<td>( G' )</td>
<td>( \frac{2\pi \sigma}{\ln(b/a)} )</td>
<td>( \frac{\pi \sigma}{\ln \left[ \left( \frac{D}{d} \right) + \sqrt{\left( \frac{D}{d} \right)^2 - 1} \right]} )</td>
<td>( \frac{\sigma w}{h} )</td>
<td>( \text{S/m} )</td>
</tr>
<tr>
<td>( C' )</td>
<td>( \frac{2\pi \varepsilon}{\ln(b/a)} )</td>
<td>( \frac{\pi \varepsilon}{\ln \left[ \left( \frac{D}{d} \right) + \sqrt{\left( \frac{D}{d} \right)^2 - 1} \right]} )</td>
<td>( \frac{\varepsilon w}{h} )</td>
<td>( \text{F/m} )</td>
</tr>
</tbody>
</table>

Notes: (1) Refer to Fig. 2-4 for definitions of dimensions. (2) \( \mu, \varepsilon, \) and \( \sigma \) pertain to the insulating material between the conductors. (3) \( R_s = \sqrt{\mu f \mu_c / \sigma_c}. \) (4) \( \mu_c \) and \( \sigma_c \) pertain to the conductors. (5) If \( \left( \frac{D}{d} \right)^2 \gg 1, \) then \( \ln \left[ \left( \frac{D}{d} \right) + \sqrt{\left( \frac{D}{d} \right)^2 - 1} \right] \approx \ln(2D/d). \)

The pertinent **constitutive parameters** apply to all three lines and consist of two groups: (1) \( \mu_c \) and \( \sigma_c \) are the magnetic permeability and electrical conductivity of the conductors, and (2) \( \varepsilon, \mu, \) and \( \sigma \) are the electrical permittivity, magnetic permeability, and electrical conductivity of the insulation material separating them.
Example 4-9: Conductance of Coaxial Cable

The radii of the inner and outer conductors of a coaxial cable of length \( l \) are \( a \) and \( b \), respectively (Fig. 4-15). The insulation material has conductivity \( \sigma \). Obtain an expression for \( G' \), the conductance per unit length of the insulation layer.

**Solution:** Let \( I \) be the total current flowing radially (along \( \hat{r} \)) from the inner conductor to the outer conductor through the insulation material. At any radial distance \( r \) from the axis of the center conductor, the area through which the current flows is \( A = 2\pi rl \). Hence,

\[
J = \hat{r} \frac{I}{A} = \hat{r} \frac{I}{2\pi rl}, \tag{4.73}
\]

and from \( J = \sigma E \),

\[
E = \hat{r} \frac{I}{2\pi \sigma rl}. \tag{4.74}
\]

In a resistor, the current flows from higher electric potential to lower potential. Hence, if \( J \) is in the \( \hat{r} \)-direction, the inner conductor must be at a higher potential than the outer conductor. Accordingly, the voltage difference between the conductors is

\[
V_{ab} = -\int_{b}^{a} \mathbf{E} \cdot d\mathbf{l} = -\int_{b}^{a} \frac{I}{2\pi \sigma l} \frac{\hat{r} \cdot \hat{r}}{r} dr
= \frac{I}{2\pi \sigma l} \ln \left( \frac{b}{a} \right). \tag{4.75}
\]

The conductance per unit length is then

\[
G' = \frac{G}{l} = \frac{1}{Rl} = \frac{I}{V_{ab}l} = \frac{2\pi \sigma}{\ln(b/a)}. \tag{4.76}
\]

\( G' = 0 \) if the insulating material is air or a perfect dielectric with zero conductivity.
Joule’s Law

Heat dissipated in a resistive conductor

For a resistor, Joule’s law is: \( \frac{dq}{dT} = P = IV \)

General case
Joule’s Law

The power dissipated in a volume containing electric field \( \mathbf{E} \) and current density \( \mathbf{J} \) is:

\[
P = \int_V \mathbf{E} \cdot \mathbf{J} \, dV \quad \text{(W)} \quad \text{(Joule’s law)}
\]

For a resistor, Joule’s law reduces to:

\[
P = I^2 R \quad \text{(W)}
\]

For a coaxial cable:

\[
P = I^2 \ln(b/a)/(2\pi\sigma l)
\]
Tech Brief 7: Resistive Sensors

An electrical sensor is a device capable of responding to an applied stimulus by generating an electrical signal whose voltage, current, or some other attribute is related to the intensity of the stimulus.

Typical stimuli: temperature, pressure, position, distance, motion, velocity, acceleration, concentration (of a gas or liquid), blood flow, etc.

Sensing process relies on measuring resistance, capacitance, inductance, induced electromotive force (emf), oscillation frequency or time delay, etc.

Figure TF7-1: Most cars use on the order of 100 sensors. (Courtesy Mercedes-Benz.)
Piezoresistivity

The Greek word *piezein* means to press.

\[ R = R_0 \left( 1 + \frac{\alpha F}{A_0} \right) \]

- \( R_0 \) = resistance when \( F = 0 \)
- \( F \) = applied force
- \( A_0 \) = cross-section when \( F = 0 \)
- \( \alpha \) = piezoresistive coefficient of material

*Figure TF7-2:* Piezoresistance varies with applied force.
Piezoresistors

**Figure TF7-3:** Piezoresistor films.

**Figure TF7-4:** Metal and silicon piezoresistors.
MicroElectroMechanical Systems, MEMS

A surface micromachined resonator fabricated by the MNX. This device can be used as both a microsensor as well as a microactuator.

From MEMS.NET
Wheatstone Bridge

Wheatstone bridge is a high sensitivity circuit for measuring small changes in resistance.

\[ V_{\text{out}} = \frac{V_0}{4} \left( \frac{\Delta R}{R_0} \right) \]

*Figure TF7-5: Wheatstone bridge circuit with piezoresistor.*
Dielectric Materials

Non-polar

Figure 4-16: In the absence of an external electric field $E$, the center of the electron cloud is co-located with the center of the nucleus, but when a field is applied, the two centers are separated by a distance $d$.

Figure 4-17: A dielectric medium polarized by an external electric field $E$. 
Polarization Field

\[ D = \varepsilon_0 E + P \]

\( P = \text{electric flux density induced by } E \)

\[ P = \varepsilon_0 \chi_e E, \quad (4.84) \]

where \( \chi_e \) is called the *electric susceptibility* of the material. Inserting Eq. (4.84) into Eq. (4.83), we have

\[ D = \varepsilon_0 E + \varepsilon_0 \chi_e E \]
\[ = \varepsilon_0 (1 + \chi_e) E = \varepsilon E, \quad (4.85) \]
Electric Breakdown

The dielectric strength $E_{ds}$ is the largest magnitude of $E$ that the material can sustain without breakdown.

**Table 4-2:** Relative permittivity (dielectric constant) and dielectric strength of common materials.

<table>
<thead>
<tr>
<th>Material</th>
<th>Relative Permittivity, $\varepsilon_r$</th>
<th>Dielectric Strength, $E_{ds}$ (MV/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air (at sea level)</td>
<td>1.0006</td>
<td>3</td>
</tr>
<tr>
<td>Petroleum oil</td>
<td>2.1</td>
<td>12</td>
</tr>
<tr>
<td>Polystyrene</td>
<td>2.6</td>
<td>20</td>
</tr>
<tr>
<td>Glass</td>
<td>4.5–10</td>
<td>25–40</td>
</tr>
<tr>
<td>Quartz</td>
<td>3.8–5</td>
<td>30</td>
</tr>
<tr>
<td>Bakelite</td>
<td>5</td>
<td>20</td>
</tr>
<tr>
<td>Mica</td>
<td>5.4–6</td>
<td>200</td>
</tr>
</tbody>
</table>

$\varepsilon = \varepsilon_r \varepsilon_0$ and $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m.
Boundary Conditions

**Figure 4-18**: Interface between two dielectric media.

\[
\mathbf{E}_{1t} = \mathbf{E}_{2t} \quad \text{(V/m).} \quad (4.90)
\]

\[
\frac{\mathbf{D}_{1t}}{\varepsilon_1} = \frac{\mathbf{D}_{2t}}{\varepsilon_2} \quad . \quad (4.91)
\]

\[
\mathbf{n}_2 \cdot (\mathbf{D}_1 - \mathbf{D}_2) = \rho_s \quad \text{(C/m²).} \quad (4.94)
\]

The normal component of \( \mathbf{D} \) changes abruptly at a charged boundary between two different media in an amount equal to the surface charge density.
Summary of Boundary Conditions

Table 4-3: Boundary conditions for the electric fields.

<table>
<thead>
<tr>
<th>Field Component</th>
<th>Any Two Media</th>
<th>Medium 1 Dielectric $\varepsilon_1$</th>
<th>Medium 2 Conductor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tangential $E$</td>
<td>$E_{1t} = E_{2t}$</td>
<td>$E_{1t} = E_{2t} = 0$</td>
<td></td>
</tr>
<tr>
<td>Tangential $D$</td>
<td>$D_{1t}/\varepsilon_1 = D_{2t}/\varepsilon_2$</td>
<td>$D_{1t} = D_{2t} = 0$</td>
<td></td>
</tr>
<tr>
<td>Normal $E$</td>
<td>$\varepsilon_1 E_{1n} - \varepsilon_2 E_{2n} = \rho_s$</td>
<td>$E_{1n} = \rho_s/\varepsilon_1$</td>
<td>$E_{2n} = 0$</td>
</tr>
<tr>
<td>Normal $D$</td>
<td>$D_{1n} - D_{2n} = \rho_s$</td>
<td>$D_{1n} = \rho_s$</td>
<td>$D_{2n} = 0$</td>
</tr>
</tbody>
</table>

Notes: (1) $\rho_s$ is the surface charge density at the boundary; (2) normal components of $E_1$, $D_1$, $E_2$, and $D_2$ are along $\hat{n}_2$, the outward normal unit vector of medium 2.

Remember $E = 0$ in a good conductor