Problem 4.36  For each of the distributions of the electric potential $V$ shown in Fig. P4.36, sketch the corresponding distribution of $E$ (in all cases, the vertical axis is in volts and the horizontal axis is in meters).

Solution:

![Graph of voltage V vs. x](image1.png)

![Graph of electric field E vs. x](image2.png)

(a)
Figure P4.36: Electric potential and corresponding electric field distributions of Problem 4.36.
**Problem 4.44**  A coaxial resistor of length $l$ consists of two concentric cylinders. The inner cylinder has radius $a$ and is made of a material with conductivity $\sigma_1$, and the outer cylinder, extending between $r = a$ and $r = b$, is made of a material with conductivity $\sigma_2$. If the two ends of the resistor are capped with conducting plates, show that the resistance between the two ends is $R = l/[\pi(\sigma_1a^2 + \sigma_2(b^2 - a^2))]$.

**Solution:** Due to the conducting plates, the ends of the coaxial resistor are each uniform at the same potential. Hence, the electric field everywhere in the resistor will be parallel to the axis of the resistor, in which case the two cylinders can be considered to be two separate resistors in parallel. Then, from Eq. (4.70),

$$\frac{1}{R} = \frac{1}{R_{\text{inner}}} + \frac{1}{R_{\text{outer}}} = \frac{\sigma_1 A_1}{l_1} + \frac{\sigma_2 A_2}{l_2} = \frac{\sigma_1 \pi a^2}{l} + \frac{\sigma_2 \pi (b^2 - a^2)}{l},$$

or

$$R = \frac{l}{\pi(\sigma_1a^2 + \sigma_2(b^2 - a^2))} \quad (\Omega).$$
Problem 4.52  Determine the force of attraction in a parallel-plate capacitor with
$A = 5 \text{ cm}^2$, $d = 2 \text{ cm}$, and $\varepsilon_r = 4$ if the voltage across it is 50 V.

Solution: From Eq. (4.131),

$$
F = -2 \frac{\varepsilon_A |E|^2}{2} = -2\varepsilon_0(5 \times 10^{-4}) \left( \frac{50}{0.02} \right)^2 = -2.55 \times 10^{-9} \text{ (N)}. 
$$
Problem 4.61  With reference to Fig. P4.61, charge $Q$ is located at a distance $d$ above a grounded half-plane located in the $x$-$y$ plane and at a distance $d$ from another grounded half-plane in the $x$-$z$ plane. Use the image method to

(a) Establish the magnitudes, polarities, and locations of the images of charge $Q$ with respect to each of the two ground planes (as if each is infinite in extent).

(b) Find the electric potential and electric field at an arbitrary point $P = (0, y, z)$.

![Figure P4.61: Charge $Q$ next to two perpendicular, grounded, conducting half-planes.](image)

Solution:

![Figure P4.61: (a) Image charges.](image)

(a) The original charge has magnitude and polarity $+Q$ at location $(0, d, d)$. Since the negative $y$-axis is shielded from the region of interest, there might as well be a conducting half-plane extending in the $-y$ direction as well as the $+y$ direction. This ground plane gives rise to an image charge of magnitude and polarity $-Q$ at location $(0, d, -d)$. In addition, since charges exist on the conducting half-plane in the $+z$ direction, an image of this conducting half-plane also appears in the $-z$ direction.
This ground plane in the $x$-$z$ plane gives rise to the image charges of $-Q$ at $(0, -d, d)$ and $+Q$ at $(0, -d, -d)$.

(b) Using Eq. (4.47) with $N = 4$,

\[
V(x,y,z) = \frac{Q}{4\pi\varepsilon} \left( \frac{1}{|kx + \bar{y}(y-d)+\bar{z}(z-d)|} - \frac{1}{|kx + \bar{y}(y+d)+\bar{z}(z-d)|} \right)
+ \frac{1}{|kx + \bar{y}(y+d)+\bar{z}(z+d)|} - \frac{1}{|kx + \bar{y}(y-d)+\bar{z}(z+d)|} \right)
= \frac{Q}{4\pi\varepsilon} \left( \frac{1}{\sqrt{x^2 + (y-d)^2 + (z-d)^2}} - \frac{1}{\sqrt{x^2 + (y+d)^2 + (z-d)^2}} \right)
+ \frac{1}{\sqrt{x^2 + (y+d)^2 + (z+d)^2}} - \frac{1}{\sqrt{x^2 + (y-d)^2 + (z+d)^2}} \right)
= \frac{Q}{4\pi\varepsilon} \left( \frac{1}{\sqrt{x^2 + y^2 - 2yd + z^2 - 2zd + 2d^2}} - \frac{1}{\sqrt{x^2 + y^2 + 2yd + z^2 - 2zd + 2d^2}} \right)
+ \frac{1}{\sqrt{x^2 + y^2 + 2yd + z^2 + 2zd + 2d^2}} - \frac{1}{\sqrt{x^2 + y^2 - 2yd + z^2 + 2zd + 2d^2}} \right) \quad (V).

From Eq. (4.51),

\[
E = -\nabla V
= \frac{Q}{4\pi\varepsilon} \left( \frac{1}{\sqrt{x^2 + (y-d)^2 + (z-d)^2}} - \frac{1}{\sqrt{x^2 + (y+d)^2 + (z-d)^2}} \right)
+ \frac{1}{\sqrt{x^2 + (y+d)^2 + (z+d)^2}} - \frac{1}{\sqrt{x^2 + (y-d)^2 + (z+d)^2}} \right)
= \frac{Q}{4\pi\varepsilon} \left( \frac{kx + \bar{y}(y-d)+\bar{z}(z-d)}{(x^2 + (y-d)^2 + (z-d)^2)^{3/2}} - \frac{kx + \bar{y}(y+d)+\bar{z}(z-d)}{(x^2 + (y+d)^2 + (z-d)^2)^{3/2}} \right)
+ \frac{kx + \bar{y}(y+d)+\bar{z}(z+d)}{(x^2 + (y+d)^2 + (z+d)^2)^{3/2}} - \frac{kx + \bar{y}(y-d)+\bar{z}(z+d)}{(x^2 + (y-d)^2 + (z+d)^2)^{3/2}} \right) \quad (V/m).
**Problem 5.4** The rectangular loop shown in Fig. P5.4 consists of 20 closely wrapped turns and is hinged along the z-axis. The plane of the loop makes an angle of 30° with the y-axis, and the current in the windings is 0.5 A. What is the magnitude of the torque exerted on the loop in the presence of a uniform field \( \mathbf{B} = \hat{y} 2.4 \, \text{T} \)? When viewed from above, is the expected direction of rotation clockwise or counterclockwise?

![Figure P5.4: Hinged rectangular loop of Problem 5.4.](image)

**Solution:** The magnetic torque on a loop is given by \( \mathbf{T} = \mathbf{m} \times \mathbf{B} \) (Eq. (5.20)), where \( \mathbf{m} = \hat{n} N A \) (Eq. (5.19)). For this problem, it is given that \( I = 0.5 \, \text{A} \), \( N = 20 \) turns, and \( A = 0.2 \, \text{m} \times 0.4 \, \text{m} = 0.08 \, \text{m}^2 \). From the figure, \( \hat{n} = -\hat{x} \cos 30° + \hat{y} \sin 30° \). Therefore, \( \mathbf{m} = \hat{n} 0.8 \, (\text{A} \cdot \text{m}^2) \) and \( \mathbf{T} = \hat{n} 0.8 \, (\text{A} \cdot \text{m}^2) \times \hat{y} 2.4 \, \text{T} = -\hat{x} 1.66 \, (\text{N} \cdot \text{m}) \). As the torque is negative, the direction of rotation is clockwise, looking from above.
**Problem 5.8** Use the approach outlined in Example 5-2 to develop an expression for the magnetic field \( \mathbf{H} \) at an arbitrary point \( P \) due to the linear conductor defined by the geometry shown in Fig. P5.8. If the conductor extends between \( z_1 = 3 \) m and \( z_2 = 7 \) m and carries a current \( I = 15 \) A, find \( \mathbf{H} \) at \( P = (2, \phi, 0) \).

![Figure P5.8: Current-carrying linear conductor of Problem 5.8.](image)

**Solution:** The solution follows Example 5-2 up through Eq. (5.27), but the expressions for the cosines of the angles should be generalized to read as

\[
\cos \theta_1 = \frac{z - z_1}{\sqrt{r^2 + (z - z_1)^2}}, \quad \cos \theta_2 = \frac{z - z_2}{\sqrt{r^2 + (z - z_2)^2}}
\]

instead of the expressions in Eq. (5.28), which are specialized to a wire centered at the origin. Plugging these expressions back into Eq. (5.27), the magnetic field is given as

\[
\mathbf{H} = \frac{I}{4\pi} \mathbf{r} \left( \frac{z - z_1}{\sqrt{r^2 + (z - z_1)^2}} - \frac{z - z_2}{\sqrt{r^2 + (z - z_2)^2}} \right).
\]

For the specific geometry of Fig. ,

\[
\mathbf{H} = \frac{15}{4\pi \times 2} \left[ \frac{0 - 3}{\sqrt{3^2 + 2^2}} - \frac{0 - 7}{\sqrt{7^2 + 2^2}} \right] = \frac{\Phi}{77.4 \times 10^{-3} \text{ (A/m)}} = \Phi 77.4 \text{ (mA/m)}.
\]
**Problem 5.12**  Two infinitely long, parallel wires are carrying 6-A currents in opposite directions. Determine the magnetic flux density at point $P$ in Fig. P5.12.

![Diagram](image)

**Figure P5.12:** Arrangement for Problem 5.12.

**Solution:**

\[
B = \frac{\mu_0 I_1}{2\pi (0.5)} + \frac{\mu_0 I_2}{2\pi (1.5)} = \frac{\mu_0}{\pi} (6 + 2) = 8 \frac{\mu_0}{\pi} \text{ (T)}.
\]
Problem 5.21  Current $I$ flows along the positive $z$-direction in the inner conductor of a long coaxial cable and returns through the outer conductor. The inner conductor has radius $a$, and the inner and outer radii of the outer conductor are $b$ and $c$, respectively.

(a) Determine the magnetic field in each of the following regions: $0 \leq r \leq a$, $a \leq r \leq b$, $b \leq r \leq c$, and $r \geq c$.

(b) Plot the magnitude of $\mathbf{H}$ as a function of $r$ over the range from $r = 0$ to $r = 10$ cm, given that $I = 10$ A, $a = 2$ cm, $b = 4$ cm, and $c = 5$ cm.

Solution:

(a) Following the solution to Example 5-5, the magnetic field in the region $r < a$,

$$\mathbf{H} = \hat{\phi} \frac{rI}{2\pi a^2},$$

and in the region $a < r < b$,

$$\mathbf{H} = \hat{\phi} \frac{I}{2\pi r}.$$

The total area of the outer conductor is $A = \pi(c^2 - b^2)$ and the fraction of the area of the outer conductor enclosed by a circular contour centered at $r = 0$ in the region $b < r < c$ is

$$\frac{\pi(r^2 - b^2)}{\pi(c^2 - b^2)} = \frac{r^2 - b^2}{c^2 - b^2}.$$

The total current enclosed by a contour of radius $r$ is therefore

$$I_{\text{enclosed}} = I \left(1 - \frac{r^2 - b^2}{c^2 - b^2}\right) = I \frac{c^2 - r^2}{c^2 - b^2},$$

and the resulting magnetic field is

$$\mathbf{H} = \hat{\phi} \frac{I_{\text{enclosed}}}{2\pi r} = \hat{\phi} \frac{I}{2\pi r} \left(\frac{c^2 - r^2}{c^2 - b^2}\right).$$

For $r > c$, the total enclosed current is zero: the total current flowing on the inner conductor is equal to the total current flowing on the outer conductor, but they are flowing in opposite directions. Therefore, $\mathbf{H} = 0$.

(b) See Fig. P5.21.
Figure P5.21: Problem 5.21.