EE 135, Winter 2013

Reading: Chapter 5, 5.1-5.8,
        Chapter 6, 6.1-6.5

Homework #5, due 2/21: Chapter 5, problems,
        5.6, 5.14, 5.19, 5.21, 5.26, 5.40

For laboratory: read lab. introduction before lab.

NOTE: Midterm# 2 is on February 28, not February 21.

Lecture 12
Midterm #2
February 28, 2013

Material covered

Chapters 3,4,5; through 2/21 lecture
Homework #3,4,5
Maxwell’s Equations

\[ \nabla \cdot \mathbf{D} = \rho_v, \]
\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \]
\[ \nabla \cdot \mathbf{B} = 0, \]
\[ \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}. \]

Under static conditions, none of the quantities appearing in Maxwell’s equations are functions of time (i.e., \( \partial / \partial t = 0 \)). This happens when all charges are permanently fixed in space, or, if they move, they do so at a steady rate so that \( \rho_v \) and \( \mathbf{J} \) are constant in time. Under these circumstances, the time derivatives of \( \mathbf{B} \) and \( \mathbf{D} \) in Eqs. (4.1b) and (4.1d) vanish, and Maxwell’s equations reduce to

**Electrostatics**

\[ \nabla \cdot \mathbf{D} = \rho_v, \quad (4.2a) \]
\[ \nabla \times \mathbf{E} = 0. \quad (4.2b) \]

**Magnetostatics**

\[ \nabla \cdot \mathbf{B} = 0, \quad (4.3a) \]
\[ \nabla \times \mathbf{H} = \mathbf{J}. \quad (4.3b) \]

Electric and magnetic fields become decoupled under static conditions.
Boundary Conditions

\[
\oint_S \mathbf{D} \cdot ds = Q \quad \Rightarrow \quad D_{1n} - D_{2n} = \rho_s. \tag{5.78}
\]

By analogy, application of Gauss’s law for magnetism, as expressed by Eq. (5.44), leads to the conclusion that

\[
\oint_S \mathbf{B} \cdot ds = 0 \quad \Rightarrow \quad B_{1n} = B_{2n}. \tag{5.79}
\]

Thus the normal component of \(\mathbf{B}\) is continuous across the boundary between two adjacent media.

\[\hat{n}_2 \times (\mathbf{H}_1 - \mathbf{H}_2) = \mathbf{J}_s.\]

Surface currents can exist only on the surfaces of perfect conductors and superconductors. Hence, at the interface between media with finite conductivities, \(\mathbf{J}_s = 0\) and

\[H_{1t} = H_{2t}. \tag{5.85}\]
Inside the solenoid:

\[ \mathbf{B} \approx \hat{z} \mu n I = \frac{\hat{z} \mu N I}{l} \]  
(long solenoid with \( l/a \gg 1 \))
Inductance

Magnetic Flux
\[ \Phi = \int_{s} B \cdot ds \quad \text{(Wb)}. \]

Flux Linkage
\[ \Lambda = N \Phi = \mu \frac{N^2}{l} I S \quad \text{(Wb)} \]

Inductance
\[ L = \frac{\Lambda}{I} \quad \text{(H)}. \]

Solenoid
\[ L = \mu \frac{N^2}{l} S \quad \text{(solenoid),} \quad \text{(5.95)} \]

and for two-conductor configurations similar to those of Fig. 5-27,
\[ L = \frac{\Lambda}{l} = \frac{\Phi}{l} = \frac{1}{l} \int_{s} B \cdot ds. \quad \text{(5.96)} \]
Figure 5-27: To compute the inductance per unit length of a two-conductor transmission line, we need to determine the magnetic flux through the area $S$ between the conductors.
Inductance of a Two Wire Transmission Line

\[ L = \frac{1}{I} \int \hat{B} \cdot ds \]

The \( \hat{B} \) fields from the 2 wires are in the same direction between the wires.

Looping up in the \( \hat{y} \) direction:

\[ \hat{B}_{up} = \hat{y} \frac{MI}{2\pi r} \]

where \( r \) = distance from center of left wire.

\[ \hat{B}_{rt} = \hat{y} \frac{MI}{2D(4r)} \]

where \( D \) = center to center separation.
Inductance of a Two Wire Transmission Line

\[ L = \frac{1}{I} \int B \cdot ds \]

The fields from the two wires are in the same direction between them:

- Looping up in the \( \hat{y} \) direction

\[ \vec{B}_{\text{top}} = \hat{y} \frac{MI}{2\pi r} \quad \text{where} \quad r = \text{distance from center of left wire} \]

\[ \vec{B}_{\text{bot}} = \hat{y} \frac{MI}{2\pi (D-r)} \quad \text{where} \quad D = \text{center to center separation} \]

Let wire radius = \( a \)

\[ \text{then with } ds = 2dr \quad (l \text{ in the length in } z) \]

\[ \frac{ds}{dt} \text{ points in } +\hat{y} \text{ direction just like } \vec{B} \]

\[ \therefore L = \frac{1}{I} \int \frac{MI}{2\pi} \left[ \frac{1}{r} + \frac{1}{D-r} \right] dr = \frac{MI}{2\pi} \int \left[ \frac{1}{r} + \frac{1}{D-r} \right] dr \]

\[ \therefore L = \frac{MI}{2\pi} \left[ 2 \ln \left( \frac{D-a}{D} \right) \right] = \frac{MI}{2\pi} \ln \left( \frac{D}{a} \right) \quad \text{of } a_{x}a_{y} \]

\[ L_{\text{total}} = \frac{L}{2} = \frac{MI}{2\pi} \left( \ln \left( \frac{D}{a} \right) \right) \]

\[ l_{\text{transm.}} = \frac{2}{\pi} \ln \left( \frac{D}{a} \right) \text{ of } 2 \text{ wire transmission line} \]
Example 5-7: Inductance of Coaxial Cable

The magnetic field in the region $S$ between the two conductors is approximately

$$B = \hat{\Phi} \frac{\mu I}{2\pi r} \quad \text{Central wire}$$

Total magnetic flux through $S$:

$$\Phi = l \int_{a}^{b} B \, dr = l \int_{a}^{b} \frac{\mu I}{2\pi r} \, dr = \frac{\mu Il}{2\pi} \ln \left( \frac{b}{a} \right)$$

Inductance per unit length:

$$L' = \frac{L}{l} = \frac{\Phi}{lI} = \frac{\mu}{2\pi} \ln \left( \frac{b}{a} \right) \quad \text{(Example 5-7)}$$

Figure 5-28: Cross-sectional view of coaxial transmission line (Example 5-7).
Magnetic Energy

Instantaneous voltage \( V \) across an inductor:
\[ V = L \frac{di}{dt} \]

Instantaneous power \( P \) is:
\[ P = iV = Li \frac{di}{dt} \]

\[ W_m = \int P \, dt = \int L \frac{di}{dt} \, dt = \int Li \\
\]
\[ W_m = \frac{1}{2} LI^2 \] magnetic energy stored in an inductor
Magnetic Energy Density

Consider the solenoid inductor.

\[ L = \mu \frac{N^2 S L}{2} \quad \text{where} \quad S = \text{sectional area} \]
\[ L = \text{length} \]
\[ N = \text{# turns} \]

Magnetic field inside solenoid,

\[ B = \mu \frac{NI}{L} \quad \Rightarrow \quad I = BL/\mu N \]

\[ W_m = \frac{1}{2} LI^2 \quad \Rightarrow \quad \frac{1}{2} \left( \mu \frac{N^2 S L}{2} \right) \left( \frac{BL}{\mu N} \right)^2 \]

\[ W_m = \frac{1}{2} \frac{B^2}{\mu} \left( \frac{L S}{2} \right) = \text{volume of solenoid} \]

So energy density \[ w_m = \frac{W_m}{\frac{L S}{2}} = \frac{1}{2} \frac{B^2}{\mu} = \frac{1}{2} \mu H^2 = w_m \]

Remember, electrostatic energy density was

\[ W_E = \frac{1}{2} E^2 \]

The energy is stored in the \( E, H \) field.
Magnetic Energy Stored in Coaxial Cable

Length = l
Inner rad. = a, outer rad. = b
Current flowing = I
Insulation permeability = \( \mu \)

In the insulation \( H = \frac{B}{\mu} = \frac{I}{2\pi r} \), where \( r \) is dist. from center.

\[ W_m = \frac{1}{2} \int_{0}^{l} B dV \]

\[ W_m = \frac{1}{2} \mu I^2 \int_{a}^{b} \left[ \frac{I}{2\pi r} \right]^2 dV \]

\[ W_m = \frac{1}{2} \mu I^2 \int_{a}^{b} \frac{I^2}{4\pi^2 r^2} dr = \frac{\mu I^2 a}{4\pi} \frac{b}{2}\ln(b) - \frac{a}{2}\ln(a) \]

\[ W_m = \frac{\mu I^2 l}{4\pi} \ln\left(\frac{b}{a}\right) = \frac{1}{2} IL^2 /\]

since \( L_{\text{inn}} = \frac{\mu I}{2\pi} \ln\left(\frac{b}{a}\right) \)

general expression
Magnetic Energy Density

Example 5-8: Magnetic Energy in a Coaxial Cable

Magnetic field in the insulating material is

$$H = \frac{B}{\mu} = \frac{I}{2\pi r}$$

The magnetic energy stored in the coaxial cable is

$$W_m = \frac{1}{2} \int \mu H^2 \, dV = \frac{\mu I^2}{8\pi^2} \int \frac{1}{r^2} \, dV$$

$$w_m = \frac{W_m}{V} = \frac{1}{2} \mu H^2 \quad (J/m^3).$$
Mutual Inductance

Figure 5-29: Magnetic field lines generated by current $I_1$ in loop 1 linking surface $S_2$ of loop 2.
Mutual Inductance

Current $I_1$ in loop 1 creates a magnetic field $B_1$.

This results in a flux through loop 2 given by

$$\Phi_{12} = \int_{S_2} B_1 \cdot ds$$

If loop 2 has $N_2$ turns then

Flux linkage through loop 2

$$\Lambda_{12} = N_2 \Phi_{12} = N_2 \int_{S_2} B_1 \cdot ds$$

Mutual inductance $L_{12}$ is then

$$L_{12} = \frac{\Lambda_{12}}{I_1} = \frac{N_2 \int_{S_2} B_1 \cdot ds}{I_1} = L_{12}$$

Figure 5.29: Magnetic field lines generated by current $I_1$ in loop 1 linking surface $S_2$ of loop 2.
Tech Brief 11: **Inductive Sensors**

LVDT can measure displacement with submillimeter precision
Proximity Sensor

Figure TF11-5: Eddy-current proximity sensor.
Chapter 5 Relationships

Maxwell’s Magnetostatics Equations

Gauss’s Law for Magnetism
\[ \nabla \cdot \mathbf{B} = 0 \quad \Rightarrow \quad \oint_{S} \mathbf{B} \cdot d\mathbf{s} = 0 \]

Ampère’s Law
\[ \nabla \times \mathbf{H} = \mathbf{J} \quad \Rightarrow \quad \oint_{C} \mathbf{H} \cdot d\mathbf{l} = I \]

Lorentz Force on Charge \( q \)
\[ \mathbf{F} = q(\mathbf{E} + \mathbf{u} \times \mathbf{B}) \]

Magnetic Force on Wire
\[ \mathbf{F}_{m} = I \oint_{C} d\mathbf{l} \times \mathbf{B} \quad (N) \]

Magnetic Torque on Loop
\[ \mathbf{T} = \mathbf{m} \times \mathbf{B} \quad (N \cdot m) \]
\[ \mathbf{m} = \mathbf{n} N I \mathbf{A} \quad (A \cdot m^2) \]

Biot–Savart Law
\[ \mathbf{H} = \frac{I}{4\pi} \oint_{l} \frac{d\mathbf{l} \times \hat{\mathbf{R}}}{R^2} \quad (A/m) \]

Magnetic Field

Infinitely Long Wire
\[ \mathbf{B} = \hat{\mathbf{z}} \frac{\mu_0 I}{2\pi r} \quad (Wb/m^2) \]

Circular Loop
\[ \mathbf{H} = \hat{\mathbf{z}} \frac{I a^2}{2(a^2 + z^2)^{3/2}} \quad (A/m) \]

Solenoid
\[ \mathbf{B} \approx \hat{\mathbf{z}} \mu_0 n I = \frac{\hat{\mathbf{z}} \mu_0 N I}{l} \quad (Wb/m^2) \]

Vector Magnetic Potential
\[ \mathbf{B} = \nabla \times \mathbf{A} \quad (Wb/m^2) \]

Vector Poisson’s Equation
\[ \nabla^2 \mathbf{A} = -\mu \mathbf{J} \]

Inductance
\[ L = \frac{\Lambda}{I} = \frac{\Phi}{I} = \frac{1}{I} \oint_{S} \mathbf{B} \cdot d\mathbf{s} \quad (H) \]

Magnetic Energy Density
\[ w_m = \frac{1}{2} \mu H^2 \quad (J/m^3) \]
6. MAXWELL’S EQUATIONS IN TIME-VARYING FIELDS
Chapter 6 Overview

## Chapter Contents

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## Objectives

Upon learning the material presented in this chapter, you should be able to:

1. Apply Faraday’s law to compute the voltage induced by a stationary coil placed in a time-varying magnetic field or moving in a medium containing a magnetic field.
2. Describe the operation of the electromagnetic generator.
3. Calculate the displacement current associated with a time-varying electric field.
4. Calculate the rate at which charge dissipates in a material with known $\varepsilon$ and $\sigma$. 
Maxwell’s Equations

In this chapter, we will examine Faraday’s and Ampère’s laws

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<td>( \nabla \cdot \mathbf{D} = \rho_v )</td>
<td>( \oint_S \mathbf{D} \cdot ds = Q ) (6.1)</td>
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<td>Faraday’s law</td>
<td>( \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} )</td>
<td>( \oint_C \mathbf{E} \cdot dl = -\oint_S \frac{\partial \mathbf{B}}{\partial t} \cdot ds ) (6.2)*</td>
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<td>Gauss’s law for magnetism</td>
<td>( \nabla \cdot \mathbf{B} = 0 )</td>
<td>( \oint_S \mathbf{B} \cdot ds = 0 ) (6.3)</td>
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<tr>
<td>Ampère’s law</td>
<td>( \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} )</td>
<td>( \oint_C \mathbf{H} \cdot dl = \int_S \left( \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \right) \cdot ds ) (6.4)</td>
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</table>

*For a stationary surface \( S \).
Faraday’s Law

Electromotive force (voltage) induced by time-varying magnetic flux:

\[ V_{\text{emf}} = -N \frac{d\Phi}{dt} = -N \frac{d}{dt} \int_S \mathbf{B} \cdot d\mathbf{s} \quad (V) \]

**Figure 6-1:** The galvanometer (predecessor of the ammeter) shows a deflection whenever the magnetic flux passing through the square loop changes with time.

Magnetic fields can produce an electric current in a closed loop, but only if the magnetic flux linking the surface area of the loop changes with time. The key to the induction process is change. Note that we can get an emf if either the field or the area changes with time. It is the change in flux that counts.
Homopolar generator
Three types of EMF

1. A time-varying magnetic field linking a stationary loop; the induced emf is then called the \textit{transformer emf}, $V_{\text{emf}}^{tr}$.

2. A moving loop with a time-varying surface area (relative to the normal component of $\mathbf{B}$) in a static field $\mathbf{B}$; the induced emf is then called the \textit{motional emf}, $V_{\text{emf}}^{m}$.

3. A moving loop in a time-varying field $\mathbf{B}$.

The total emf is given by

$$V_{\text{emf}} = V_{\text{emf}}^{tr} + V_{\text{emf}}^{m}, \quad (6.7)$$
Stationary Loop in Time-Varying $B$  

It is important to remember that $B_{\text{ind}}$ serves to oppose the change in $B(t)$, and not necessarily $B(t)$ itself.

\[ V_{\text{emf}}^{\text{tr}} = -N \int_{s} \frac{\partial B}{\partial t} \cdot ds \]  

(transformer emf),

The connection between the direction of $ds$ and the polarity of $V_{\text{emf}}^{\text{tr}}$ is governed by the following right-hand rule: if $ds$ points along the thumb of the right hand, then the direction of the contour $C$ indicated by the four fingers is such that it always passes across the opening from the positive terminal of $V_{\text{emf}}^{\text{tr}}$ to the negative terminal.

\[ I = \frac{V_{\text{emf}}^{\text{tr}}}{R + R_1}. \]  

(6.9)

For good conductors, $R_1$ usually is very small, and it may be ignored in comparison with practical values of $R$.

The polarity of $V_{\text{emf}}^{\text{tr}}$ and hence the direction of $I$ is governed by Lenz's law, which states that the current in the loop is always in a direction that opposes the change of magnetic flux $\Phi(t)$ that produced $I$.

Figure 6-2: (a) Stationary circular loop in a changing magnetic field $B(t)$, and (b) its equivalent circuit.
Lenz’s Law

The polarity of the transformer EMF and thus the direction of the induced current flow is always to oppose the change in magnetic flux.
Example 6-1: Inductor in a Changing Magnetic Field

An inductor is formed by winding $N$ turns of a thin conducting wire into a circular loop of radius $a$. The inductor loop is in the $x$-$y$ plane with its center at the origin, and connected to a resistor $R$, as shown in Fig. 6-3. In the presence of a magnetic field $\mathbf{B} = B_0(\hat{y}2 + \hat{z}3) \sin \omega t$, where $\omega$ is the angular frequency, find

(a) the magnetic flux linking a single turn of the inductor,
(b) the transformer emf, given that $N = 10$, $B_0 = 0.2$ T, $a = 10$ cm, and $\omega = 10^3$ rad/s,
(c) the polarity of $V_{\text{emf}}^{\text{tr}}$ at $t = 0$, and
(d) the induced current in the circuit for $R = 1$ k$\Omega$ (assume the wire resistance to be much smaller than $R$).

Figure 6-3: Circular loop with $N$ turns in the $x$-$y$ plane. The magnetic field is $\mathbf{B} = B_0(\hat{y}2 + \hat{z}3) \sin \omega t$ (Example 6-1).
Example 6-1 solution

Solution: (a) The magnetic flux linking each turn of the inductor is

\[
\Phi = \int_{S} \mathbf{B} \cdot d\mathbf{s}
\]

\[
= \int_{S} \left[ B_0 (\hat{y} \cdot \hat{z}) \sin \omega t \right] \cdot \hat{z} \, ds
\]

\[
= 3\pi a^2 B_0 \sin \omega t.
\]

(b) To find \( V_{\text{emf}}^{\text{tr}} \), we can apply Eq. (6.8) or we can apply the general expression given by Eq. (6.6) directly. The latter approach gives

\[
V_{\text{emf}}^{\text{tr}} = -N \frac{d\Phi}{dt}
\]

\[
= -\frac{d}{dt} \left( 3\pi N a^2 B_0 \sin \omega t \right)
\]

\[
= -3\pi N \omega a^2 B_0 \cos \omega t.
\]

For \( N = 10, \ a = 0.1 \text{ m}, \ \omega = 10^3 \text{ rad/s}, \) and \( B_0 = 0.2 \text{ T}, \)

\[
V_{\text{emf}}^{\text{tr}} = -188.5 \cos 10^3 t \quad (V).
\]

(c) At \( t = 0, \ d\Phi/dt > 0 \) and \( V_{\text{emf}}^{\text{tr}} = -188.5 \text{ V}. \) Since the flux is increasing, the current \( I \) must be in the direction shown in Fig. 6-3 in order to satisfy Lenz’s law. Consequently, terminal 2 is at a higher potential than terminal 1 and

\[
V_{\text{emf}}^{\text{tr}} = V_1 - V_2
\]

\[
= -188.5 \quad (V).
\]

(d) The current \( I \) is given by

\[
I = \frac{V_2 - V_1}{R}
\]

\[
= \frac{188.5 \cos 10^3 t}{10^3} = 0.19 \cos 10^3 t \quad (A).
\]
CD Module 6.1 Circular Loop in Time-varying Magnetic Field

Faraday's law of induction is demonstrated by simulating the current induced in a loop in response to the change in magnetic flux flowing through it.

Module 6.1

Circular Loop in Time-varying Magnetic Field

Demonstration of Faraday's Law

The circular wire loop shown in the figure is connected to a simple circuit composed of a resistor $R_L$ in series with a current meter. The time-varying magnetic flux linking the surface of the loop induces an emf and hence a current through $R$. The purpose of this demo is to illustrate, in the form of a slow-motion video, how the current $I$ varies with time, in both magnitude and direction, when $B(t) = B_0 \cos \omega t$.

Note that $I(t)$ is a maximum when the slope of $B(t)$ is a maximum, which occurs when $B$ itself is zero. The direction of $I(t)$ is dictated by Lenz's Law.
Example 6-2: Lenz’s Law

Determine voltages $V_1$ and $V_2$ across the 2-$\Omega$ and 4-$\Omega$ resistors shown in Fig. 6-4. The loop is located in the $x$–$y$ plane, its area is 4 m$^2$, the magnetic flux density is $\mathbf{B} = -0.3t$ (T), and the internal resistance of the wire may be ignored.

Solution: The flux flowing through the loop is

$$\Phi = \int_{S} \mathbf{B} \cdot d\mathbf{s} = \int_{S} (-0.3t) \cdot \hat{z} \, ds$$

$$= -0.3t \times 4 = -1.2t \quad \text{(Wb)},$$

and the corresponding transformer emf is

$$V_{\text{emf}}^{\text{tr}} = -\frac{d\Phi}{dt} = 1.2 \quad \text{(V)}.$$

The total voltage of 1.2 V is distributed across two resistors in series. Consequently,

$$I = \frac{V_{\text{emf}}^{\text{tr}}}{R_1 + R_2}$$

$$= \frac{1.2}{2 + 4} = 0.2 \text{ A},$$

and

$$V_1 = IR_1 = 0.2 \times 2 = 0.4 \text{ V},$$

$$V_2 = IR_2 = 0.2 \times 4 = 0.8 \text{ V}.$$
Ideal Transformer

\[ V_1 = -N_1 \frac{d\Phi}{dt}. \]

A similar relation holds true on the secondary side:

\[ V_2 = -N_2 \frac{d\Phi}{dt}. \]

\[ \frac{V_1}{V_2} = \frac{N_1}{N_2} \quad \frac{I_1}{I_2} = \frac{N_2}{N_1} \]

\[ R_{in} = \frac{V_1}{I_1} \]

\[ R_{in} = \frac{V_2}{I_2} \left( \frac{N_1}{N_2} \right)^2 = \left( \frac{N_1}{N_2} \right)^2 R_L. \]  \hspace{1cm} (6.20)

When the load is an impedance \( Z_L \) and \( V_1 \) is a sinusoidal source, the phasor-domain equivalent of Eq. (6.20) is

\[ Z_{in} = \left( \frac{N_1}{N_2} \right)^2 Z_L. \]  \hspace{1cm} (6.21)

Figure 6-5: In a transformer, the directions of \( I_1 \) and \( I_2 \) are such that the flux \( \Phi \) generated by one of them is opposite to that generated by the other. The direction of the secondary winding in (b) is opposite to that in (a), and so are the direction of \( I_2 \) and the polarity of \( V_2 \).
Motional EMF

Magnetic force on charge $q$ moving with velocity $\mathbf{u}$ in a magnetic field $\mathbf{B}$:

$$\mathbf{F}_m = q(\mathbf{u} \times \mathbf{B}).$$

This magnetic force is equivalent to the electrical force that would be exerted on the particle by the electric field $\mathbf{E}_m$ given by

$$\mathbf{E}_m = \frac{\mathbf{F}_m}{q} = \mathbf{u} \times \mathbf{B}.$$ 

This, in turn, induces a voltage difference between ends 1 and 2, with end 2 being at the higher potential. The induced voltage is

$$V_{\text{emf}}^m = V_{12} = \int_{2}^{1} \mathbf{E}_m \cdot d\mathbf{l} = \int_{2}^{1} (\mathbf{u} \times \mathbf{B}) \cdot d\mathbf{l}.$$ 

For the conducting wire, $\mathbf{u} \times \mathbf{B} = \mathbf{\hat{x}} u \times \mathbf{\hat{z}} B_0 = -\mathbf{\hat{y}} u B_0$ and $d\mathbf{l} = \mathbf{\hat{y}} dl$. Hence,

$$V_{\text{emf}}^m = V_{12} = -u B_0 l. \quad (6.25)$$
Motional EMF

In general, if any segment of a closed circuit with contour $C$ moves with a velocity $\mathbf{u}$ across a static magnetic field $\mathbf{B}$, then the induced motional emf is given by

$$V_{\text{emf}}^m = \oint_C (\mathbf{u} \times \mathbf{B}) \cdot d\mathbf{l} \quad \text{(motional emf)}. \quad (6.26)$$

*Only those segments of the circuit that cross magnetic field lines contribute to $V_{\text{emf}}^m$.***
Example 6-3: Sliding Bar

The length of the loop is related to \( u \) by \( x_0 = ut \). Hence

\[
V_{\text{emf}}^m = V_{12} = V_{43} = \int_{3}^{4} (\mathbf{u} \times \mathbf{B}) \cdot d\mathbf{l} \\
= \int_{3}^{4} (\hat{\mathbf{x}}u \times \hat{\mathbf{z}}B_0x_0) \cdot \hat{\mathbf{y}} \ dl = -uB_0x_0l.
\]

Note that \( \mathbf{B} \) increases with \( x \)

\[
\mathbf{B} = \hat{\mathbf{z}}B_0x
\]

The length of the loop is related to \( u \) by \( x_0 = ut \). Hence

\[
V_{\text{emf}}^m = -B_0u^2lt \quad (V).
\]
Example 6-5: Moving Rod Next to a Wire

The wire shown in Fig. 6-10 carries a current $I = 10$ A. A 30-cm-long metal rod moves with a constant velocity $u = \hat{z}5$ m/s. Find $V_{12}$.

$$B = \hat{\phi} \frac{\mu_0 I}{2\pi r}$$

$$V_{12} = \int_{40 \text{ cm}}^{10 \text{ cm}} (u \times B) \cdot dl$$

$$= \int_{40 \text{ cm}}^{10 \text{ cm}} \left( \hat{z}5 \times \hat{\phi} \frac{\mu_0 I}{2\pi r} \right) \cdot \hat{r} \, dr$$

$$= -\frac{5 \mu_0 I}{2\pi} \int_{40 \text{ cm}}^{10 \text{ cm}} \frac{dr}{r}$$

$$= -\frac{5 \times 4\pi \times 10^{-7} \times 10}{2\pi} \times \ln \left(\frac{40}{10}\right)$$

$$= 13.9 \text{ (\muV).}$$
DC Homopolar Generator (Faraday)

rotate metal disk in constant $\vec{B}$ field.

$\vec{F} = q\vec{N} \times \vec{B}$

$\vec{F} = qN IB$ free motion

pushing down to

rim of disk

:: potential diff. between center

and rim

$V_{\text{emp}} = -\int \vec{E} \cdot d\vec{r} = -\int E \, dr$

but $E = \frac{F}{q} = NB = \omega BR$

:: $V_{\text{emp}} = -\int \omega BR \, dr = \frac{1}{2} \omega BR^2 = V_{\text{emp}}$
EM Motor/Generator Reciprocity

Motor: Electrical to mechanical energy conversion

Generator: Mechanical to electrical energy conversion
EM Generator

As the loop rotates with an angular velocity $\omega$ about its own axis, segment 1–2 moves with velocity $u$ given by

$$u = \hat{n}\omega \frac{w}{2}$$

Also:

$$\hat{n} \times \hat{z} = \hat{x} \sin \alpha$$

Segment 3-4 moves with velocity $-u$. Hence:

$$V_{\text{emf}}^m = V_{14} = \int_{l/2}^{1} (u \times B) \cdot dl + \int_{3}^{4} (u \times B) \cdot dl$$

$$= \int_{-l/2}^{l/2} \left[ \left(\hat{n}\omega \frac{w}{2}\right) \times \hat{z}B_0 \right] \cdot \hat{x} \, dx$$

$$+ \int_{l/2}^{-l/2} \left[ \left(-\hat{n}\omega \frac{w}{2}\right) \times \hat{z}B_0 \right] \cdot \hat{x} \, dx.$$
EM generator

Just use Faraday's law:

\[ \Phi = \oint \mathbf{B} \cdot d\mathbf{l} = \oint Z \mathbf{E} \cdot d\mathbf{s} \]

\[ = B_0 A \omega (\omega t + \phi) \quad \text{, where } A = WL \text{ area of loop} \]

\[ \omega = \omega t + \phi \]

\[ \therefore V_{emf} = -\frac{d\Phi}{dt} = -\frac{d}{dt} [B_0 A \omega (\omega t + \phi)] \]

\[ V_{emf} = A W B_0 \omega \sin(\omega t + \phi) \]
CD Module 6.2 Rotating Wire Loop in Constant Magnetic Field

The principle of the electromagnetic generator is demonstrated by a rectangular loop rotating in the presence of a magnetic field.

Demonstration of Motional EMF

A rectangular wire loop of area \( A \) rotates at an angular frequency \( \omega \) in a constant magnetic flux density \( B_0 \). The purpose of the demo is to illustrate how the current varies in time relative to the loop's position.

Note the direction of the current and its magnitude, as indicated by its brightness.

\[ I_{\text{max}} = \omega B_0 A \]
Tech Brief 12: EMF Sensors

Piezoelectric crystals generate a voltage across them proportional to the compression or tensile (stretching) force applied across them.

Piezoelectric transducers are used in medical ultrasound, microphones, loudspeakers, accelerometers, etc.

Piezoelectric crystals are bidirectional: pressure generates emf, and conversely, emf generates pressure (through shape distortion).
Faraday Accelerometer

Figure TF12-3: In a Faraday accelerometer, the induced emf is directly proportional to the velocity of the loop (into and out of the magnet’s cavity).

The acceleration $\mathbf{a}$ is determined by differentiating the velocity $\mathbf{u}$ with respect to time.
The Thermocouple

The thermocouple measures the unknown temperature $T_2$ at a junction connecting two metals with different thermal conductivities, relative to a reference temperature $T_1$.

In today’s temperature sensor designs, an artificial cold junction is used instead. The artificial junction is an electric circuit that generates a voltage equal to that expected from a reference junction at temperature $T_1$. 

Figure TF12-4: Principle of the thermocouple.
Displacement Current

Ampère’s law in differential form is given by

\[ \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \quad \text{(Ampère’s law).} \]  \hspace{1cm} (6.41)

Integrating both sides of Eq. (6.41) over an arbitrary open surface \( S \) with contour \( C \), we have

\[ \int_S (\nabla \times \mathbf{H}) \cdot d\mathbf{s} = \int_S \mathbf{J} \cdot d\mathbf{s} + \int_S \frac{\partial \mathbf{D}}{\partial t} \cdot d\mathbf{s}. \]  \hspace{1cm} (6.42)

This term is conduction current \( I_c \)

This term must represent a current

Application of Stokes’s theorem gives:

\[ \int_C \mathbf{H} \cdot d\mathbf{l} = I_c + \int_S \frac{\partial \mathbf{D}}{\partial t} \cdot d\mathbf{s} \quad \text{(Ampère’s law)} \]

Cont.
Displacement Current

Define the displacement current as:

\[ I_d = \int_S \mathbf{J}_d \cdot ds = \int_S \frac{\partial \mathbf{D}}{\partial t} \cdot ds, \quad (6.44) \]

where \( \mathbf{J}_d = \frac{\partial \mathbf{D}}{\partial t} \) represents a displacement current density. In view of Eq. (6.44),

\[ \oint_C \mathbf{H} \cdot dl = I_c + I_d = I, \quad (6.45) \]

The displacement current does not involve real charges; it is an equivalent current that depends on \( \frac{\partial \mathbf{D}}{\partial t} \).
Capacitor Circuit

**Given:** Wires are perfect conductors and capacitor insulator material is perfect dielectric.

**For Surface \( S_1 \):**

\[ I_1 = I_{1c} + I_{1d} \]

\[ I_{1c} = C \frac{dV_C}{dt} = C \frac{d}{dt} (V_0 \cos \omega t) = -CV_0 \omega \sin \omega t \]

\[ I_{1d} = 0 \quad (D = 0 \text{ in perfect conductor}) \]

**For Surface \( S_2 \):**

\[ I_2 = I_{2c} + I_{2d} \]

\[ I_{2c} = 0 \quad (\text{perfect dielectric}) \]

\[ E = \hat{y} \frac{V_c}{d} = \hat{y} \frac{V_0}{d} \cos \omega t \]

\[ I_{2d} = \int_{S} \frac{\partial D}{\partial t} \cdot ds \]

\[ = \int_{A} \left[ \frac{\partial}{\partial t} \left( \hat{y} \frac{\varepsilon V_0}{d} \cos \omega t \right) \right] \cdot (\hat{y} \, ds) \]

\[ = -\frac{\varepsilon A}{d} V_0 \omega \sin \omega t = -CV_0 \omega \sin \omega t \]

**Conclusion:** \( I_1 = I_2 \)
Example 6-7: Displacement Current Density

The conduction current flowing through a wire with conductivity \( \sigma = 2 \times 10^7 \) S/m and relative permittivity \( \varepsilon_r = 1 \) is given by \( I_c = 2 \sin \omega t \) (mA). If \( \omega = 10^9 \) rad/s, find the displacement current.

Solution: The conduction current \( I_c = J A = \sigma E A \), where \( A \) is the cross section of the wire. Hence,

\[
E = \frac{I_c}{\sigma A} = \frac{2 \times 10^{-3} \sin \omega t}{2 \times 10^7 A} = \frac{1 \times 10^{-10}}{A} \sin \omega t \quad (V/m).
\]

Application of Eq. (6.44), with \( D = \varepsilon E \), leads to

\[
I_d = J_d A = \varepsilon A \frac{\partial E}{\partial t} = \varepsilon A \frac{\partial}{\partial t} \left( \frac{1 \times 10^{-10}}{A} \sin \omega t \right)
\]

where we used \( \omega = 10^9 \) rad/s and \( \varepsilon = \varepsilon_0 = 8.85 \times 10^{-12} \) F/m. Note that \( I_c \) and \( I_d \) are in phase quadrature (90° phase shift between them). Also, \( I_d \) is about nine orders of magnitude smaller than \( I_c \), which is why the displacement current usually is ignored in good conductors.

\[
= \varepsilon \omega \times 10^{-10} \cos \omega t = 0.885 \times 10^{-12} \cos \omega t \quad (A),
\]
### Boundary Conditions

**Table 6-2:** Boundary conditions for the electric and magnetic fields.

<table>
<thead>
<tr>
<th>Field Components</th>
<th>General Form</th>
<th>Medium 1 Dielectric</th>
<th>Medium 2 Dielectric</th>
<th>Medium 1 Dielectric</th>
<th>Medium 2 Conductor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tangential E</td>
<td>( \hat{n}_2 \times (E_1 - E_2) = 0 )</td>
<td>( E_{1t} = E_{2t} )</td>
<td></td>
<td></td>
<td>( E_{1t} = E_{2t} = 0 )</td>
</tr>
<tr>
<td>Normal D</td>
<td>( \hat{n}_2 \cdot (D_1 - D_2) = \rho_s )</td>
<td>( D_{1n} - D_{2n} = \rho_s )</td>
<td>( D_{1n} = \rho_s )</td>
<td>( D_{2n} = 0 )</td>
<td></td>
</tr>
<tr>
<td>Tangential H</td>
<td>( \hat{n}_2 \times (H_1 - H_2) = J_s )</td>
<td>( H_{1t} = H_{2t} )</td>
<td>( H_{1t} = J_s )</td>
<td>( H_{2t} = 0 )</td>
<td></td>
</tr>
<tr>
<td>Normal B</td>
<td>( \hat{n}_2 \cdot (B_1 - B_2) = 0 )</td>
<td>( B_{1n} = B_{2n} )</td>
<td>( B_{1n} = B_{2n} = 0 )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: (1) \( \rho_s \) is the surface charge density at the boundary; (2) \( J_s \) is the surface current density at the boundary; (3) normal components of all fields are along \( \hat{n}_2 \), the outward unit vector of medium 2; (4) \( E_{1t} = E_{2t} \) implies that the tangential components are equal in magnitude and parallel in direction; (5) direction of \( J_s \) is orthogonal to \( (H_1 - H_2) \).
Charge Current Continuity Equation

Current I out of a volume is equal to rate of decrease of charge Q contained in that volume:

\[ I = -\frac{dQ}{dt} = -\frac{d}{dt} \int_V \rho_v \, dV \]

\[ \oint_S J \cdot ds = -\frac{d}{dt} \int_V \rho_v \, dV \]

\[ \oint_S J \cdot ds = \int_V \nabla \cdot J \, dV = -\frac{d}{dt} \int_V \rho_v \, dV \]

Figure 6-14: The total current flowing out of a volume \( V \) is equal to the flux of the current density \( J \) through the surface \( S \), which in turn is equal to the rate of decrease of the charge enclosed in \( V \).

\[ \nabla \cdot J = -\frac{\partial \rho_v}{\partial t} \quad , \quad (6.54) \]

which is known as the \textit{charge-current continuity relation}, or simply the \textit{charge continuity equation}. Used Divergence Theorem
Question 1: What happens if you place a certain amount of free charge inside of a material?  
Answer: The charge will move to the surface of the material, thereby returning its interior to a neutral state.

Question 2: How fast will this happen?  
Answer: It depends on the material; in a good conductor, the charge dissipates in less than a femtosecond, whereas in a good dielectric, the process may take several hours.

Derivation of charge density equation:

\[ \nabla \cdot \mathbf{J} = -\frac{\partial \rho_v}{\partial t}. \quad (6.58) \]

In a conductor, the point form of Ohm’s law, given by Eq. (4.63), states that \( \mathbf{J} = \sigma \mathbf{E} \). Hence,

\[ \sigma \nabla \cdot \mathbf{E} = -\frac{\partial \rho_v}{\partial t}. \quad (6.59) \]

Next, we use Eq. (6.1), \( \nabla \cdot \mathbf{E} = \rho_v/\varepsilon \), to obtain the partial differential equation

\[ \frac{\partial \rho_v}{\partial t} + \frac{\sigma}{\varepsilon} \rho_v = 0. \quad (6.60) \]
Solution of Charge Dissipation Equation

\[ \frac{\partial \rho_v}{\partial t} + \frac{\sigma}{\varepsilon} \rho_v = 0 \]

Given that \( \rho_v = \rho_{vo} \) at \( t = 0 \), the solution of Eq. (6.60) is

\[ \rho_v(t) = \rho_{vo} e^{-(\sigma/\varepsilon)t} = \rho_{vo} e^{-t/\tau_r} \quad (\text{C/m}^3), \]

where \( \tau_r = \varepsilon/\sigma \) is called the relaxation time constant.

For copper: \( \tau_r = 1.53 \times 10^{-19} \) s

For mica: \( \tau_r = 5.31 \times 10^4 \) s = 15 hours
EM Potentials

Static condition

\[ V(R) = \frac{1}{4\pi \varepsilon} \int_{V'} \frac{\rho_v(R_i)}{R'} \, dV' \]

Dynamic condition

\[ V(R, t) = \frac{1}{4\pi \varepsilon} \int_{V'} \frac{\rho_v(R_i, t)}{R'} \, dV' \]

Dynamic condition with propagation delay:  Similarly, for the magnetic vector potential:

\[ V(R, t) = \frac{1}{4\pi \varepsilon} \int_{V'} \frac{\rho_v(R_i, t - R'/u_p)}{R'} \, dV' \quad (V) \]

\[ A(R, t) = \frac{\mu}{4\pi} \int_{V'} \frac{J(R_i, t - R'/u_p)}{R'} \, dV' \quad (Wb/m) \]

Figure 6-16: Electric potential \( V(R) \) due to a charge distribution \( \rho_v \) over a volume \( V' \).
Time Harmonic Potentials

If charges and currents vary sinusoidally with time:

\[ \rho_v(\mathbf{R}_i, t) = \rho_v(\mathbf{R}_i) \cos(\omega t + \phi) \]

we can use phasor notation:

\[ \rho_v(\mathbf{R}_i, t) = \Re \left[ \tilde{\rho}_v(\mathbf{R}_i) e^{j\omega t} \right], \]

with

\[ \tilde{\rho}_v(\mathbf{R}_i) = \rho_v(\mathbf{R}_i) e^{j\phi}. \]

Expressions for potentials become:

\[ \widetilde{V}(\mathbf{R}) = \frac{1}{4\pi \varepsilon} \int_{V'} \frac{\tilde{\rho}_v(\mathbf{R}_i) e^{-jkr}}{R'} \, dV' \quad \text{(V).} \]

\[ \widetilde{A}(\mathbf{R}) = \frac{\mu}{4\pi} \int_{V'} \frac{\tilde{J}(\mathbf{R}_i) e^{-jkr}}{R'} \, dV', \]

Also:

\[ \mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t} \quad \text{(dynamic case).} \]

\[ \tilde{H} = \frac{1}{\mu} \nabla \times \tilde{A}. \]

Maxwell’s equations become:

\[ \nabla \times \tilde{\mathbf{E}} = -j\omega \mu \tilde{\mathbf{H}} \]

or

\[ \tilde{\mathbf{H}} = -\frac{1}{j\omega \mu} \nabla \times \tilde{\mathbf{E}}. \]

\[ \nabla \times \tilde{\mathbf{H}} = j\omega \varepsilon \tilde{\mathbf{E}} \quad \text{or} \quad \tilde{\mathbf{E}} = \frac{1}{j\omega \varepsilon} \nabla \times \tilde{\mathbf{H}}. \]

\[ k = \frac{\omega}{u_p} \]
Example 6-8: Relating $E$ to $H$

In a nonconducting medium with $\varepsilon = 16\varepsilon_0$ and $\mu = \mu_0$, the electric field intensity of an electromagnetic wave is

$$E(z, t) = \hat{x} 10 \sin(10^{10}t - kz) \quad \text{(V/m)}. \quad (6.88)$$

Determine the associated magnetic field intensity $H$ and find the value of $k$.

**Solution:** We begin by finding the phasor $\tilde{E}(z)$ of $E(z, t)$. Since $E(z, t)$ is given as a sine function and phasors are defined in this book with reference to the cosine function, we rewrite Eq. (6.88) as

$$E(z, t) = \hat{x} 10 \cos(10^{10}t - kz - \pi/2) \quad \text{(V/m)}$$

$$= \Re\left[\tilde{E}(z) e^{j\omega t}\right], \quad (6.89)$$

with $\omega = 10^{10}$ (rad/s) and

$$\tilde{E}(z) = \hat{x} 10e^{-jkz} e^{-j\pi/2} = -\hat{x} j 10 e^{-jkz}. \quad (6.90)$$
To find both $\hat{H}(z)$ and $k$, we will perform a “circle”: we will use the given expression for $\hat{E}(z)$ in Faraday’s law to find $\hat{H}(z)$; then we will use $\hat{H}(z)$ in Ampère’s law to find $\hat{E}(z)$, which we will then compare with the original expression for $\hat{E}(z)$; and the comparison will yield the value of $k$. Application of Eq. (6.87) gives

$$\hat{H}(z) = -\frac{1}{j\omega\mu} \nabla \times \hat{E}$$

$$= -\frac{1}{j\omega\mu} \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial / \partial x & \partial / \partial y & \partial / \partial z \\ -j10e^{-jkz} & 0 & 0 \end{vmatrix}$$

$$= -\frac{1}{j\omega\mu} \left[ \hat{y} \frac{\partial}{\partial z} (-j10e^{-jkz}) \right]$$

$$= -\hat{y}j \frac{10k}{\omega\mu} e^{-jkz}. \quad (6.91)$$
Example 6-8 cont.

So far, we have used Eq. (6.90) for $\tilde{E}(z)$ to find $\tilde{H}(z)$, but $k$ remains unknown. To find $k$, we use $\tilde{H}(z)$ in Eq. (6.86) to find $\tilde{E}(z)$:

$$\tilde{E}(z) = \frac{1}{j\omega\varepsilon} \nabla \times \tilde{H}$$

$$= \frac{1}{j\omega\varepsilon} \left[ -\hat{x} \frac{\partial}{\partial z} \left( -j \frac{10k}{\omega\mu} e^{-jkz} \right) \right]$$

$$= -\hat{x} j \frac{10k^2}{\omega^2 \mu \varepsilon} e^{-jkz}.$$  \hspace{1cm} (6.92)

Equating Eqs. (6.90) and (6.92) leads to

$$k^2 = \omega^2 \mu \varepsilon,$$

or

$$k = \omega \sqrt{\mu \varepsilon}$$

$$= 4 \omega \sqrt{\mu_0 \varepsilon_0}$$

$$= \frac{4 \omega}{c} = \frac{4 \times 10^{10}}{3 \times 10^8} = 133 \text{ (rad/m).}$$  \hspace{1cm} (6.93)
Example 6-8 cont.

With $k$ known, the instantaneous magnetic field intensity is then given by

$$
\mathbf{H}(z, t) = \Re \left[ \hat{\mathbf{H}}(z) e^{j \omega t} \right]
= \Re \left[ -\hat{y} j \frac{10k}{\omega \mu} e^{-jkz} e^{j \omega t} \right]
= \hat{y} 0.11 \sin(10^{10} t - 133z) \quad \text{(A/m).} \quad (6.94)
$$

We note that $k$ has the same expression as the phase constant of a lossless transmission line [Eq. (2.49)].
Summary

Chapter 6 Relationships

Faraday’s Law
\[ V_{\text{emf}} = -\frac{d\Phi}{dt} = -\frac{d}{dt} \int_S \mathbf{B} \cdot d\mathbf{s} = V_{\text{emf}}^{\text{tr}} + V_{\text{emf}}^{\text{in}} \]

Transformer
\[ V_{\text{emf}}^{\text{tr}} = -N \int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{s} \quad (N \text{ loops}) \]

Motional
\[ V_{\text{emf}}^{\text{in}} = \oint_C (\mathbf{u} \times \mathbf{B}) \cdot d\mathbf{l} \]

Charge-Current Continuity
\[ \nabla \cdot \mathbf{J} = -\frac{\partial \rho_v}{\partial t} \]

EM Potentials
\[ \mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t} \]
\[ \mathbf{B} = \nabla \times \mathbf{A} \]

Current Density
Conduction
\[ \mathbf{J}_c = \sigma \mathbf{E} \]
Displacement
\[ \mathbf{J}_d = \frac{\partial \mathbf{D}}{\partial t} \]

Conductor Charge Dissipation
\[ \rho_v(t) = \rho_{vo} e^{-\left(\frac{\alpha}{\varepsilon}\right)t} = \rho_{vo} e^{-t/\tau} \]
Midterm #2
February 28, 2013

*Material covered*

Chapters 3, 4, 5; through 2/21 lecture
Homework #3, 4, 5